Developmental Mathematics
Chapter 15 Review

Objective [15.1a] Find the principal square roots and their opposites of the whole numbers from 0^2 to 25^2.

Brief Procedure	Example	Practice Exercise
The principal square root of a number n, denoted $\sqrt{n}$, is the positive square root of the number. The opposite of the principal square root of a number n is denoted $-\sqrt{n}$.	Find each of the following. a) $\sqrt{144}$ b) $-\sqrt{49}$ a) The number 144 has two square roots, 12 and -12. The notation $\sqrt{144}$ denotes the principal, or positive, square root so $\sqrt{144} = 12$. b) $\sqrt{49}$ represents the positive square root of 49, or 7, and $-\sqrt{49}$ represents the opposite of that number. Thus, $-\sqrt{49} = -7$.	1. Find $\sqrt{81}$. A. -81 B. 81 C. -9 D. 9

Objective [15.1b] Approximate square roots of real numbers using a calculator.

Brief Procedure	Example	Practice Exercise
Use a calculator with a square-root key. Round to the desired number of decimal places.	Use a calculator to approximate each of the following square roots to three decimal places. a) $\sqrt{17}$ b) $-\sqrt{104.2}$ a) Using a calculator with a 10-digit readout, we get $\sqrt{17} \approx 4.123105626 \approx 4.123$. b) Using a calculator with a 10-digit readout, we get $-\sqrt{104.2} \approx -10.20784012 \approx -10.208$.	2. Use a calculator to approximate $-\sqrt{70}$ to three decimal places. A. -8.367 B. -8.366 C. 8.366 D. 8.367

Objective [15.1c] Solve applied problems involving square roots.

Brief Procedure	Example	Practice Exercise
Substitute in a formula and carry out the resulting computation using a calculator to approximate square roots if needed.	The formula $r = 2\sqrt{5L}$ can be used to approximate the speed r, in miles per hour, of a car that has left a skid mark of length L, in feet. What was the speed of a car that left skid marks of length 120 ft? We substitute 120 for L and find an approximation. $r = 2\sqrt{5L} = 2\sqrt{5 \cdot 120} = 2\sqrt{600} \approx 48.990$ The speed of the car was about 49.0 mph.	3. Use the formula $r = 2\sqrt{5L}$ to find the speed of a car that left skid marks of length 85 ft. A. About 92.2 mph B. About 53.6 mph C. About 41.2 mph D. About 38.8 mph

Objective [15.1d] Identify radicands of radical expressions.

Brief Procedure	Example	Practice Exercise
In a radical expression, the radicand is the expression under the radical.	Identify the radicand in the expression $2\sqrt{x-1}$. The radicand is the expression under the radical, $x-1$.	4. Identify the radicand in the expression $x\sqrt{3}$. A. x B. 3 C. $3x$ D. $-\dfrac{3}{x}$

Objective [15.1e] Identify whether a radical expression represents a real number.

Brief Procedure	Example	Practice Exercise
Determine whether the radicand is positive or negative. Radical expressions with negative radicands do not represent real numbers (are not meaningful as real numbers).	Determine whether each expression is meaningful as a real number. a) $-\sqrt{23}$ b) $\sqrt{-23}$ a) The radicand, 23, is positive so $-\sqrt{23}$ is meaningful as a real number. b) The radicand, -23, is negative so $\sqrt{-23}$ is not meaningful as a real number.	5. Determine whether $-\sqrt{-5}$ is meaningful as a real number. A. Yes B. No

Objective [15.1f] Simplify a radical expression with a perfect-square radicand.

Brief Procedure	Example	Practice Exercise
For any real number A, $\sqrt{A^2} = \|A\|$. (That is, for any real number A, the principal square root of A^2 is the absolute value of A.) For any nonnegative real number A, $\sqrt{A^2} = A$. (That is, for any nonnegative real number A, the principal square root of A^2 is A.	Simplify each of the following. Assume that expressions under radicals represent nonnegative real numbers. a) $\sqrt{(xy)^2}$ b) $\sqrt{16y^2}$ c) $\sqrt{x^2-4x+4}$ a) $\sqrt{(xy)^2} = xy$, since xy is assumed to be nonnegative. b) $\sqrt{16y^2} = 4y$, since $4y$ is assumed to be nonnegative. c) $\sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = x-2$, since $x-2$ is assumed to be nonnegative.	6. Simplify. Assume that the expression under the radical represents a nonnegative real number. $\sqrt{(5a)^2}$ A. $25a^2$ B. $5a^2$ C. $5a$ D. $\sqrt{5a}$

Objective [15.2a] Simplify radical expressions.		
Brief Procedure	**Example**	**Practice Exercise**
A square-root radical expression is simplified when its radicand has no factors that are perfect squares. To simplify, factor and use the fact that $\sqrt{AB} = \sqrt{A}\sqrt{B}$.	Simplify $\sqrt{12b^2}$ by factoring. $\sqrt{12b^2} = \sqrt{4 \cdot 3 \cdot b^2} = \sqrt{4}\sqrt{b^2}\sqrt{3} = 2b\sqrt{3}$	7. Simplify $\sqrt{45y}$ by factoring. A. $5\sqrt{3y}$ B. $y\sqrt{45}$ C. $3y\sqrt{5}$ D. $3\sqrt{5y}$

Objective [15.2b] Simplify radical expressions where radicands are powers.		
Brief Procedure	**Example**	**Practice Exercise**
Find the square root of an even power by taking half the exponent. If an odd power occurs, first express the power in terms of the largest even power. Then simplify the even power.	Simplify: a) $\sqrt{n^6}$ b) $\sqrt{50x^{17}}$ a) $\sqrt{n^6} = \sqrt{(n^3)^2} = n^3$ (Note that the exponent in the result, 3, is half the original exponent, 6. That is, $\frac{1}{2} \cdot 6 = 3$.) b) $\sqrt{50x^{17}} = \sqrt{25 \cdot 2 \cdot x^{16} \cdot x} = \sqrt{25}\sqrt{x^{16}}\sqrt{2x} = 5x^8\sqrt{2x}$	8. Simplify $\sqrt{y^{13}}$. A. y^6 B. $y^6\sqrt{y}$ C. y^7 D. $y^7\sqrt{y}$

Objective [15.2c] Multiply radical expressions and simplify, if possible.		
Brief Procedure	**Example**	**Practice Exercise**
For any nonnegative radicands A and B, $\sqrt{A} \cdot \sqrt{B} = \sqrt{A \cdot B}$. Use this rule to multiply radical expressions. Then use it in reverse to simplify the product: $\sqrt{AB} = \sqrt{A}\sqrt{B}$.	Multiply and simplify: $\sqrt{2x^3}\sqrt{10x^2}$. $\sqrt{2x^3}\sqrt{10x^2} = \sqrt{2x^3 \cdot 10x^2} = \sqrt{2 \cdot x^2 \cdot x \cdot 2 \cdot 5 \cdot x^2} = \sqrt{2 \cdot 2}\sqrt{x^2}\sqrt{x^2}\sqrt{5x} = 2 \cdot x \cdot x \cdot \sqrt{5x} = 4x^2\sqrt{5x}$	9. Multiply and simplify: $\sqrt{6x}\sqrt{3x^5}$. A. $3x^2\sqrt{2x}$ B. $x^3\sqrt{18}$ C. $3x^2\sqrt{2}$ D. $3x^3\sqrt{2}$

Objective [15.3a] Divide radical expressions.		
Brief Procedure	**Example**	**Practice Exercise**
For any nonnegative number A and any positive number B, $\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$. (The quotient of two square roots is the square root of the quotient of the radicands.)	Divide and simplify: $\frac{\sqrt{15a^4}}{\sqrt{3a}}$. $\frac{\sqrt{15a^4}}{\sqrt{3a}} = \sqrt{\frac{15a^4}{3a}} = \sqrt{5a^3} = \sqrt{5 \cdot a^2 \cdot a} = \sqrt{a^2} \cdot \sqrt{5a} = a\sqrt{5a}$	10. Divide and simplify: $\frac{\sqrt{56y^5}}{\sqrt{8y}}$. A. $y^2\sqrt{7}$ B. $7y^2$ C. $\sqrt{7y^4}$ D. $y^4\sqrt{7}$

Objective [15.3b] Simplify square roots of quotients.

Brief Procedure	Example	Practice Exercise
For any nonnegative number A and any positive number B, $\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$. (We can take the square roots of the numerator and the denominator separately.)	Simplify: $\sqrt{\frac{50}{90}}$. $\sqrt{\frac{50}{90}} = \sqrt{\frac{25 \cdot 2}{49 \cdot 2}} = \sqrt{\frac{25}{49} \cdot \frac{2}{2}}$ $= \sqrt{\frac{25}{49} \cdot 1} = \sqrt{\frac{25}{49}}$ $= \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$	11. Simplify: $\sqrt{\frac{x^2}{36}}$. A. $\frac{x}{36}$ B. $\frac{x^2}{6}$ C. $\frac{x}{6}$ D. $6x$

Objective [15.3c] Rationalize the denominator of a radical expression.

Brief Procedure	Example	Practice Exercise
Method 1. Multiply by 1 under the radical to make the denominator a perfect square. **Method 2.** Multiply by 1 outside the radical to make the denominator a perfect square.	Rationalize the denominator. a) $\sqrt{\frac{5}{6}}$ b) $\frac{\sqrt{x}}{\sqrt{3}}$ a) We use method 1, choosing 6/6 for 1. $\sqrt{\frac{5}{6}} = \sqrt{\frac{5}{6} \cdot \frac{6}{6}}$ $= \sqrt{\frac{30}{36}} = \frac{\sqrt{30}}{\sqrt{36}}$ $= \frac{\sqrt{30}}{6}$ b) We use method 2, choosing $\sqrt{3}/\sqrt{3}$ for 1. $\frac{\sqrt{x}}{\sqrt{3}} = \frac{\sqrt{x}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{3x}}{\sqrt{9}} = \frac{\sqrt{3x}}{3}$	12. Rationalize the denominator: $\frac{\sqrt{5}}{\sqrt{11}}$. A. $\frac{\sqrt{5}}{11}$ B. $\frac{\sqrt{55}}{11}$ C. $\frac{5}{11}$ D. $\frac{5}{\sqrt{11}}$

Objective [15.4a] Add or subtract with radical notation, using the distributive law to simplify.

Brief Procedure	Example	Practice Exercise
Use the distributive laws to collect like radicals (terms with the same radicand). Sometimes radical terms must be simplified before like radicals can be identified.	Add: $5\sqrt{2} + 4\sqrt{18}$. $5\sqrt{2} + 4\sqrt{18} = 5\sqrt{2} + 4\sqrt{9 \cdot 2}$ $= 5\sqrt{2} + 4\sqrt{9}\sqrt{2}$ $= 5\sqrt{2} + 4 \cdot 3\sqrt{2}$ $= 5\sqrt{2} + 12\sqrt{2}$ $= (5+12)\sqrt{2}$ $= 17\sqrt{2}$	13. Subtract: $6\sqrt{3} - \sqrt{48}$ A. $2\sqrt{3}$ B. $4\sqrt{3}$ C. $8\sqrt{3}$ D. $10\sqrt{3}$

Objective [15.4b] Multiply expressions involving radicals, where some of the expressions contain more than one term.		
Brief Procedure	**Example**	**Practice Exercise**
Use the distributive laws and special products of polynomials.	Multiply: $(2-\sqrt{5})(3+4\sqrt{5})$. We use FOIL. $(2-\sqrt{5})(3+4\sqrt{5})$ $= 2\cdot 3 + 2\cdot 4\sqrt{5} - \sqrt{5}\cdot 3 - \sqrt{5}\cdot 4\sqrt{5}$ $= 6 + 8\sqrt{5} - 3\sqrt{5} - 4\cdot 5$ $= 6 + 8\sqrt{5} - 3\sqrt{5} - 20$ $= -14 + 5\sqrt{5}$	14. Multiply: $(\sqrt{a}-7)^2$. A. $a-49$ B. a^2-49 C. $a-14\sqrt{a}+49$ D. $a^2-14\sqrt{a}+49$

Objective [15.4c] Rationalize denominators having two terms.		
Brief Procedure	**Example**	**Practice Exercise**
Multiply by 1 using the conjugate of the denominator for the numerator and denominator of the expression for 1. (Some examples of conjugates are $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$, $c+\sqrt{d}$ and $c-\sqrt{d}$, and $\sqrt{m}-n$ and $\sqrt{m}+n$.)	Rationalize the denominator: $\dfrac{6}{\sqrt{5}-\sqrt{3}}$. $\dfrac{6}{\sqrt{5}-\sqrt{3}} = \dfrac{6}{\sqrt{5}-\sqrt{3}}\cdot\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ $= \dfrac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$ $= \dfrac{6\sqrt{5}+6\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$ $= \dfrac{6\sqrt{5}+6\sqrt{3}}{5-3}$ $= \dfrac{6\sqrt{5}+6\sqrt{3}}{2}$ $= \dfrac{2(3\sqrt{5}+3\sqrt{3})}{2\cdot 1}$ $= \dfrac{2}{2}\cdot\dfrac{3\sqrt{5}+3\sqrt{3}}{1}$ $= 3\sqrt{5}+3\sqrt{3}$	15. Rationalize the denominator: $\dfrac{3}{1-\sqrt{x}}$. A. $\dfrac{3}{1+\sqrt{x}}$ B. $\dfrac{3+3\sqrt{x}}{1-x}$ C. $\dfrac{3}{1-x}$ D. $\dfrac{3}{1+x}$

Objective [15.5a] Solve radical equations with one or two radical terms isolated, using the principle of squaring once.		
Brief Procedure	Example	Practice Exercise
First isolate a radical. Square both sides of the equation to eliminate radicals and then solve for the variable. When the principle of squaring is used to solve an equation, the possible solutions must be checked in the original equation.	Solve: $x = 3 + \sqrt{x-1}$. First we subtract 3 on both sides to isolate the radical. $x = 3 + \sqrt{x-1}$ $x - 3 = \sqrt{x-1}$ $(x-3)^2 = (\sqrt{x-1})^2$ $x^2 - 6x + 9 = x - 1$ $x^2 - 7x + 10 = 0$ $(x-2)(x-5) = 0$ $x - 2 = 0$ *or* $x - 5 = 0$ $x = 2$ *or* $x = 5$ We check each possible solution. Check: For 2: $x = 3 + \sqrt{x-1}$ $2 \ ? \ 3 + \sqrt{2-1}$ $3 + \sqrt{1}$ $3 + 1$ 4 FALSE For 5: $x = 3 + \sqrt{x-1}$ $5 \mid 3 + \sqrt{5-1}$ $3 + \sqrt{4}$ $3 + 2$ 5 TRUE The number 5 checks, but 2 does not. Thus, the solution is 5.	16. Solve: $\sqrt{3x+7} = \sqrt{4x+1}$. A. 6 B. 8 C. $6, 8$ D. No solution

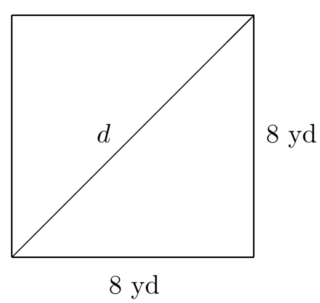
Objective [15.5b] Solve radical equations with two radical terms, using the principle of squaring twice.

Brief Procedure	Example	Practice Exercise
1. Isolate one of the radical terms. 2. Square both sides of the equation. 3. Isolate the remaining radical term. 4. Square both sides of the equation again. 5. Solve the equation. 6. Check the possible solutions.	Solve: $1 = \sqrt{x+9} - \sqrt{x}$. $1 = \sqrt{x+9} - \sqrt{x}$ $\sqrt{x} + 1 = \sqrt{x+9}$ $(\sqrt{x}+1)^2 = (\sqrt{x+9})^2$ $x + 2\sqrt{x} + 1 = x + 9$ $2\sqrt{x} = 8$ $\sqrt{x} = 4$ $(\sqrt{x})^2 = 4^2$ $x = 16$ The number 16 checks. It is the solution.	17. Solve: $\sqrt{x+3} - \sqrt{x-2} = 1$. A. 1 B. 4 C. 6 D. No solution

Objective [15.5c] Solve applied problems using radical equations.

Brief Procedure	Example	Practice Exercise
Substitute in a formula and then solve the resulting radical equation.	At a height of h meters, a person can see V kilometers to the horizon, where $V = 3.5\sqrt{h}$. Martin can see 53.2 km to the horizon from the top of a cliff. What is the altitude of Martin's eyes? We substitute 53.2 for V in the equation $V = 3.5\sqrt{h}$ and solve for h. $53.2 = 3.5\sqrt{h}$ $\frac{53.2}{3.5} = \sqrt{h}$ $15.2 = \sqrt{h}$ $(15.2)^2 = (\sqrt{h})^2$ $231.04 = h$ The altitude of Martin's eyes is about 231 m.	18. A passenger can see 301 km to the horizon through an airplane window. What is the altitude of the passenger's eyes? A. 1053.5 m B. 3529 m C. 5226.4 m D. 7396 m

Objective [15.6a] Given the lengths of any two sides of a right triangle, find the length of the third side.		
Brief Procedure	Example	Practice Exercise
In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$. The equation $a^2 + b^2 = c^2$ is called the Pythagorean equation.	In a right triangle, find the length of the side not given. Give an exact answer and an approximation to three decimal places. $a = 5,\ c = 9$ We substitute in the Pythagorean equation. $a^2 + b^2 = c^2$ $5^2 + b^2 = 9^2$ $25 + b^2 = 81$ $b^2 = 56$ $b = \sqrt{56}$ $b \approx 7.483$	19. In a right triangle, find the length of the side not given. $b = 12,\ c = 13$ A. 1 B. 5 C. 7 D. 9

Objective [15.6b] Solve applied problems involving right triangles.		
Brief Procedure	Example	Practice Exercise
Use the Pythagorean equation.	Find the length of a diagonal of a square whose sides are 8 yd long. We first make a drawing. We label the diagonal d.  We know that $8^2 + 8^2 = d^2$. We solve this equation. $8^2 + 8^2 = d^2$ $64 + 64 = d^2$ $128 = d^2$ $\sqrt{128} = d$ $11.314 \approx d$ The length of a diagonal is $\sqrt{128} \approx 11.314$ yd.	20. How long is a guy wire reaching from the top of a 10-ft pole to a point on the ground 7 ft from the pole? A. About 7.141 ft B. About 10.049 ft C. About 12.207 ft D. About 15.811 ft