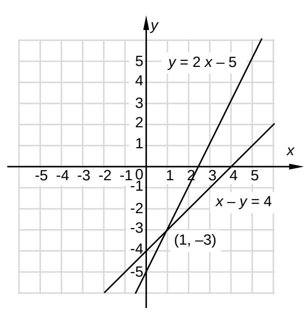
Developmental Mathematics
Chapter 14 Review

Objective [14.1a] Determine whether an ordered pair is a solution of a system of two equations.		
Brief Procedure	**Example**	**Practice Exercise**
Determine whether the ordered pair is a solution of both equations. If it is, it is a solution of the system of equations.	Determine whether $(1,-1)$ is a solution of the system of equations $y = x - 2,$ $2x + y = 3.$ Using alphabetical order of the variables, we substitute 1 for x and -1 for y in both equations. $\begin{array}{r\|l} \multicolumn{2}{c}{y = x - 2} \\ \hline -1 \;?\; & 1 - 2 \\ & -1 \quad \text{TRUE} \end{array}$ $\begin{array}{r\|l} \multicolumn{2}{c}{2x + y = 3} \\ \hline 2 \cdot 1 + (-1) \;?\; & 3 \\ 2 - 1 & \\ 1 & \quad \text{FALSE} \end{array}$ The pair $(1,-1)$ is not a solution of $2x + y = 3$, so it is not a solution of the system of equations.	1. Determine whether $(2,-3)$ is a solution of the system of equations $2x - y = 7,$ $x = y + 5.$ A. Yes B. No

Objective [14.1b] Solve systems of two linear equations in two variables by graphing.		
Brief Procedure	**Example**	**Practice Exercise**
Graph both equations and find the coordinates of the point(s) of intersection, if any exist. If the graphs are parallel lines, there is no point of intersection and, hence, no solution. If the equations have the same graph, there are infinitely many points of intersection and, thus, infinitely many solutions. Otherwise, there is exactly one point of intersection and, hence, exactly one solution.	Solve this system of equations by graphing: $x - y = 4,$ $y = 2x - 5.$ We graph the equations.  The point of intersection appears to be $(1,-3)$. This checks in both equations, so it is the solution.	2. Solve this system of equations by graphing: $3x - 2y = 6,$ $x - y = 1.$ A. $(-2,-6)$ B. $(2,0)$ C. $(4,3)$ D. $(5,4)$

Objective [14.2a] Solve a system of equations in two variables by the substitution method when one of the equations has a variable alone on one side.

Brief Procedure	Example	Practice Exercise
Using the equation with a variable alone on one side, substitute for that variable in the other equation, obtaining an equation in one variable. Solve that equation; then substitute in either original equation to find the other variable.	Solve the system $x + y = -2,$ (1) $x = 2y + 7.$ (2) First substitute $2y + 7$ for x in Equation (1) and solve for y. $x + y = -2$ $(2y + 7) + y = -2$ $3y + 7 = -2$ $3y = -9$ $y = -3$ Now substitute -3 for y in either of the original equations and find x. We choose Equation (2) because it has x alone on one side. $x = 2y + 7 = 2(-3) + 7 = -6 + 7 = 1$ The ordered pair $(1, -3)$ checks in both equations, so it is the solution of the system of equations.	3. Solve the system $y = x - 2,$ $x - 2y = 6.$ A. The y-value is 0. B. The y-value is -12. C. The y-value is -2. D. The y-value is -4.

Objective [14.2b] Solve a system of two equations in two variables by the substitution method when neither equation has a variable alone on one side.

Brief Procedure	Example	Practice Exercise
Solve one equation for one of the variables, choosing a variable that has a coefficient of 1, if possible. Then substitute for that variable in the other equation, obtaining an equation in one variable. Solve that equation. Finally, substitute in either original equation to find the other variable.	Solve the system $x - 2y = 1,$ (1) $2x - 3y = 3.$ (2) We solve Equation (1) for x, since the coefficient of x is 1 in that equation. $x - 2y = 1$ $x = 2y + 1$ (3) Now substitute for x in Equation (2) and solve for y. $2x - 3y = 3$ $2(2y + 1) - 3y = 3$ $4y + 2 - 3y = 3$ $y + 2 = 3$ $y = 1$ Now substitute 1 for y in Equation (1), (2), or (3) and find x. We choose Equation (3) since it is already solved for x. $x = 2y + 1 = 2 \cdot 1 + 1 = 2 + 1 = 3$ The ordered pair $(3, 1)$ checks in both equations, so it is the solution of the system of equations.	4. Solve the system $x + y = 3,$ $5x + 2y = 3.$ A. The x-value is -1. B. The x-value is 4. C. The x-value is -3. D. The x-value is 1.

Objective [14.2c] Solve applied problems by translating to a system of two equations and then solving using the substitution method.

Brief Procedure	Example	Practice Exercise

Brief Procedure

Use the five-step problem solving process.

Example

The sum of two numbers is 1. One number is 11 more than the other. Find the numbers.

1. *Familiarize.* We let $x =$ the smaller number and $y =$ the larger number.
2. *Translate.* The first statement gives us one equation.

$$\begin{array}{ccc} \underbrace{\text{The sum of two numbers}} & \text{is} & 1. \\ \downarrow & \downarrow & \downarrow \\ x+y & = & 1 \end{array}$$

Now we translate the second statement.

$$\begin{array}{cccccc} \underbrace{\text{One number}} & \text{is} & 11 & \underbrace{\text{more than}} & \underbrace{\text{the other.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & = & 11 & + & x \end{array}$$

We now have a system of equations:

$$x + y = 1, \qquad (1)$$
$$y = 11 + x. \qquad (2)$$

3. *Solve.* First we substitute $11 + x$ for y in Equation (1).

$$\begin{aligned} x + (11 + x) &= 1 \\ 2x + 11 &= 1 \\ 2x &= -10 \\ x &= -5 \end{aligned}$$

Now substitute -5 for x in Equation (2).

$$y = 11 + x = 11 + (-5) = 6$$

4. *Check.* The sum of -5 and 6 is 1. Also, 6 is 11 more than -5, so the numbers check.
5. *State.* The numbers are -5 and 6.

Practice Exercise

5. The perimeter of a rectangular poster is 12 ft. The length is twice the width. Find the length and width.

A. The length is 2 ft.

B. The length is 4 ft.

C. The length is 6 ft.

D. The length is 8 ft.

Objective [14.3a] Solve a system of two equations in two variables using the elimination method when no multiplication is necessary.

Brief Procedure	Example	Practice Exercise
Add the corresponding sides of the equations to eliminate a variable. Solve for that variable. Then substitute in either of the original equations to find the other variable.	Solve the system $2x - y = 5, \quad (1)$ $x + y = 7. \quad (2)$ First, we add. $\begin{array}{rl} 2x - y &= 5 \\ x + y &= 7 \\ \hline 3x \quad &= 12 \\ x &= 4 \end{array}$ Now substitute 4 for x in either of the original equations and solve for y. We use Equation (2). $x + y = 7$ $4 + y = 7$ $y = 3$ The ordered pair (4,3) checks in both equations, so it is a solution of the system of equations.	6. Solve the system $3x + 2y = 1,$ $x - 2y = -13.$ A. The y-value is -3. B. The y-value is -1. C. The y-value is 5. D. The y-value is 8.

Objective [14.3b] Solve a system of two equations in two variables using the elimination method when multiplication is necessary.

Brief Procedure	Example	Practice Exercise
Multiply one or both equations by appropriate constants to find equivalent equations with a pair of terms that are opposites. Then add the corresponding sides of the equations to eliminate a variable. Solve for that variable. Finally, substitute in either of the original equations to find the other variable.	Solve the system $2a - 3b = 7, \quad (1)$ $3a - 2b = 8. \quad (2)$ We could eliminate either a or b. Here we decide to eliminate the a-terms. Multiply Equation (1) by 3 and Equation (2) by -2. Then add and solve for b. $\begin{array}{rl} 6a - 9b &= 21 \\ -6a + 4b &= -16 \\ \hline -5b &= 5 \\ b &= -1 \end{array}$ Next substitute -1 for b in either of the original equations. $2a - 3b = 7 \quad (1)$ $2a - 3(-1) = 7$ $2a + 3 = 7$ $2a = 4$ $a = 2$ The ordered pair $(2, -1)$ checks in both equations, so it is a solution of the system of equations.	7. Solve the system $3x + 2y = 5,$ $x - y = 5.$ A. The y-value is -4. B. The y-value is -2. C. The y-value is 2. D. The y-value is 3.

Objective [14.3c] Solve applied problems by translating to a system of two equations and then solving using the elimination method.		
Brief Procedure	**Example**	**Practice Exercise**
Use the five-step problem solving process.	Two angles are supplementary. (Supplementary angles are angles whose sum is 180°.) The difference between twice one angle and the other angle is 30°. Find the angles. 1. *Familiarize.* We let x and y represent the angles. 2. *Translate.* We know that the sum of the angles is 180°. This gives us one equation. $\underbrace{\text{The sum of the angles}}_{x+y}\ \underbrace{\text{is}}_{=}\ \underbrace{180°.}_{180}$ We use the additional information given to translate to a second equation. $\underbrace{\text{Twice one angle}}_{2x}\ \underbrace{\text{less}}_{-}\ \underbrace{\text{the other}}_{y}\ \underbrace{\text{is}}_{=}\ \underbrace{30°.}_{30}$ We now have a system of equations: $x + y = 180$ $2x - y = 30.$ 3. *Solve.* First we add the equations to eliminate the y-terms. $x + y = 180$ $2x - y = 30$ $3x \quad = 210$ $x = 70$ Now we substitute in one of the original equations to find y. We use the first equation. $x + y = 180$ $70 + y = 180$ $y = 110$ 4. *Check.* The sum of 70° and 110° is 180°. Also, $2 \cdot 70° - 110° = 140° - 110° = 30°$, so the answer checks. 5. *State.* The angles are 70° and 110°.	8. The sum of two numbers is −3. The sum of twice one number and the other is 4. Find the numbers. A. One number is −10. B. One number is −7. C. One number is −4. D. One number is 4.

Objective [14.4a] Solve applied problems by translating to a system of two equations in two variables.

Brief Procedure	Example
Use the five-step problem solving process.	See below.

Example

Solution A is 40% acid and solution B is 55% acid. How much of each should be used in order to make 100 L of a solution that is 46% acid?

1. *Familiarize.* Let x and y represent the number of liters of 40% and 55% solution to be used, respectively. We organize the given information in a table.

Type of solution	A	B	Mixture
Amount of solution	x	y	100 L
Percent of acid	40%	55%	46%
Amount of acid in solution	$40\%x$	$55\%y$	$46\% \times 100$, or 46 L

2. *Translate.* The first row of the table gives us one equation.

$$x + y = 100$$

The last row gives us a second equation.

$$40\%x + 55\%y = 46, \text{ or}$$
$$0.4x + 0.55y = 46$$

After multiplying by 100 on both sides of the second equation to clear decimals, we have the following system of equations.

$$x + y = 100, \quad (1)$$
$$40x + 55y = 4600 \quad (2)$$

3. *Solve.* We use the elimination method. First multiply Equation (1) by -40 and then add to eliminate the x-terms.

$$\begin{aligned} -40x - 40y &= -4000 \\ 40x + 55y &= 4600 \\ \hline 15y &= 600 \\ y &= 40 \end{aligned}$$

Now substitute in Equation (2) and solve for x.

$$\begin{aligned} x + y &= 100 \\ x + 40 &= 100 \\ x &= 60 \end{aligned}$$

4. *Check.* The sum of 60 and 40 is 100. Also, 40% of 60 L is 24 L and 55% of 40 L is 22 L. These add up to 46 L, so the answer checks.

5. *State.* 60 L of solution A and 40 L of solution B should be used.

Practice Exercise

9. There were 220 tickets sold for a school play. The price for students was \$3 and it was \$7 for non-students. A total of \$1080 was collected. How many of each type of ticket were sold?

A. Student: 75, non-student: 145
B. Student: 90, non-student: 130
C. Student: 95, non-student: 125
D. Student: 115, non-student: 105

Objective [14.5a] Solve motion problems using the formula $d = rt$.

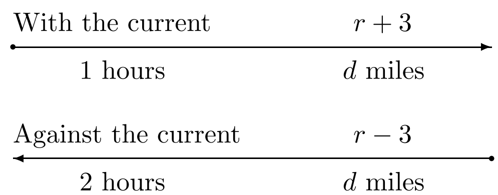
Brief Procedure

Use the five-step problem solving process. It is often convenient to translate to a system of equations.

Example

A canoeist paddled for 1 hr with a 3 mph current. The return trip against the current took 2 hr. Find the speed of the canoe in still water.

1. *Familiarize.* We first make a drawing. Let d = the distance traveled in one direction and let r = the speed of the canoe in still water.When the canoe travels with the current, its speed is $r + 3$ and traveling against the current the speed is $r - 3$.



   We can also organize the given information in a table.

   $d \quad = \quad r \quad \cdot \quad t$

	Distance	Speed	Time
With current	d	$r + 3$	1
Against current	d	$r - 3$	2

2. *Translate.* Using $d = rt$, we get an equation from each row of the table.

   $d = (r + 3)1, \qquad (1)$
   $d = (r - 3)2 \qquad (2)$

3. *Solve.* We use the substitution method, substituting $(r - 3)2$ for d in Equation (1).

$$\begin{aligned} (r-3)2 &= (r+3)1 \\ 2r - 6 &= r + 3 \\ r - 6 &= 3 \\ r &= 9 \end{aligned}$$

4. *Check.* When $r = 9$, then $r + 3 = 12$ and $12 \cdot 1 = 12$, the distance traveled with the current. When $r = 9$, then $r - 3 = 6$ and $6 \cdot 2 = 12$, the distance traveled against the current. Since the distances are the same, the answer checks.

5. *State.* The speed of the canoe in still water is 9 mph.

Practice Exercise

10. A train leaves a station and travels west at 80 mph. One hour later a second train leaves the same station and travels west on a parallel track at 100 mph. When will it overtake the first train?

A. 4 hr after the first train leaves
B. 5 hr after the first train leaves
C. 6 hr after the first train leaves
D. 8 hr after the first train leaves