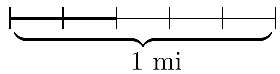
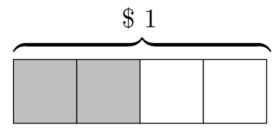
Developmental Mathematics
Chapter 2 Review

Objective [2.1a] Write fractional notation for part of an object or part of a set of objects.

Brief Procedure	Example	Practice Exercise
Determine the number of parts into which the object is divided and then determine how many parts are taken or shaded.	What part is shaded?  The object is divided into 5 parts of the same size and 2 of them are shaded. Thus, $2\cdot\frac{1}{5}$, or $\frac{2}{5}$ of the object is shaded.	1. What part is shaded?  A. $\frac{1}{4}$ B. $\frac{2}{4}$ C. $\frac{3}{4}$ D. $\frac{2}{2}$

Objective [2.1b] Simplify fractional notation like n/n to 1, $0/n$ to 0, and $n/1$ to n.

Brief Procedure	Example	Practice Exercise
For any whole number n that is not 0, $\frac{n}{n}=1$ and $\frac{0}{n}=0$. For any whole number n, $\frac{n}{1}=n$.	Simplify $\frac{6}{6}$, $\frac{0}{10}$, and $\frac{3}{1}$. $\frac{6}{6}=1,\ \frac{0}{10}=0,\ \frac{3}{1}=3$	2. Simplify (a) $\frac{5}{1}$, (b) $\frac{12}{12}$, and (c) $\frac{0}{2}$. A. (a) 5, (b) 1, (c) 0 B. (a) 1, (b) 12, (c) 0 C. (a) 5, (b) 1, (c) 2 D. (a) 5, (b) 12, (c) 2

Objective [2.1c] Multiply using fractional notation.

Brief Procedure	Example	Practice Exercise
Multiply the numerators to get the new numerator; multiply the denominators to get the new denominator.	Multiply: $\frac{3}{4}\cdot\frac{5}{2}$. $\frac{3}{4}\cdot\frac{5}{2}=\frac{3\cdot 5}{4\cdot 2}=\frac{15}{8}$	3. Multiply: $\frac{5}{8}\cdot\frac{7}{6}$. A. $\frac{12}{14}$ B. $\frac{35}{14}$ C. $\frac{35}{48}$ D. $\frac{30}{56}$

Objective [2.1d] Find another name for a number, but having a new denominator. Use multiplying by 1.		
Brief Procedure	Example	Practice Exercise
Ask: What number n should we multiply the denominator by in order to get the new denominator? Then multiply the fraction by 1 using n/n.	Find a name for $\frac{2}{3}$ with a denominator of 12. Since $3 \cdot 4 = 12$, we multiply by $\frac{4}{4}$: $\frac{2}{3} = \frac{2}{3} \cdot \frac{4}{4} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$	4. Find a name for $\frac{3}{4}$ with a denominator of 20. A. $\frac{3}{20}$ B. $\frac{8}{20}$ C. $\frac{15}{20}$ D. $\frac{19}{20}$

Objective [2.1e] Simplify fractional notation.		
Brief Procedure	Example	Practice Exercise
Remove a factor of 1 to get the name for the fraction that has the smallest numerator and denominator.	Simplify: $\frac{16}{36}$. $\frac{16}{36} = \frac{4 \cdot 4}{4 \cdot 9} = \frac{4}{4} \cdot \frac{4}{9} = 1 \cdot \frac{4}{9} = \frac{4}{9}$	5. Simplify: $\frac{9}{24}$. A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{9}{8}$

Objective [2.2a] Multiply and simplify using fractional notation.		
Brief Procedure	Example	Practice Exercise
a) Write the products in the numerator and the denominator, but do not carry out the products. b) Factor the numerator and the denominator. c) Factor the fraction to remove factors of 1. d) Carry out the remaining products.	Multiply and simplify: $\frac{3}{4} \cdot \frac{2}{9}$. $\frac{3}{4} \cdot \frac{2}{9} = \frac{3 \cdot 2}{4 \cdot 9} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 2 \cdot 3 \cdot 3} =$ $\frac{3 \cdot 2}{3 \cdot 2} \cdot \frac{1}{2 \cdot 3} = 1 \cdot \frac{1}{2 \cdot 3} = \frac{1}{2 \cdot 3} = \frac{1}{6}$	6. Multiply and simplify: $\frac{5}{6} \cdot \frac{4}{15}$. A. $\frac{2}{9}$ B. $\frac{3}{7}$ C. $\frac{4}{18}$ D. $\frac{20}{90}$

Objective [2.2b] Find the reciprocal of a number.		
Brief Procedure	Example	Practice Exercise
Interchange the numerator and the denominator.	Find the reciprocals of $\frac{5}{9}$, 3, and $\frac{1}{6}$. Interchange the numerator and denominator of each fraction. The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$; the reciprocal of 3, or $\frac{3}{1}$, is $\frac{1}{3}$; the reciprocal of $\frac{1}{6}$ is $\frac{6}{1}$, or 6.	7. Find the reciprocal of 13. A. 0 B. $\frac{1}{13}$ C. $\frac{1}{3}$ D. 13

Objective [2.2c] Divide and simplify using fractional notation.		
Brief Procedure	Example	Practice Exercise
Multiply the dividend by the reciprocal of the divisor. Then simplify.	Divide and simplify: $\frac{5}{4} \div \frac{25}{16}$. $\frac{5}{4} \div \frac{25}{16} = \frac{5}{4} \cdot \frac{16}{25} = \frac{5 \cdot 16}{4 \cdot 25} = \frac{5 \cdot 4 \cdot 4}{4 \cdot 5 \cdot 5} =$ $\frac{5 \cdot 4}{5 \cdot 4} \cdot \frac{4}{5} = \frac{4}{5}$	8. Divide and simplify: $\frac{2}{3} \div \frac{8}{9}$. A. $\frac{3}{4}$ B. $\frac{5}{6}$ C. $\frac{4}{3}$ D. $\frac{16}{27}$

Objective [2.2d] Solve equations of the type $a \cdot x = b$ and $x \cdot a = b$, where a and b may be fractions.		
Brief Procedure	Example	Practice Exercise
Divide by a on both sides of the equation.	Solve: $\frac{5}{3} \cdot y = \frac{20}{9}$. $\frac{5}{3} \cdot y = \frac{20}{9}$ $y = \frac{20}{9} \div \frac{5}{3}$ $y = \frac{20}{9} \cdot \frac{3}{5}$ $y = \frac{20 \cdot 3}{9 \cdot 5} = \frac{4 \cdot 5 \cdot 3}{3 \cdot 3 \cdot 5}$ $y = \frac{5 \cdot 3}{5 \cdot 3} \cdot \frac{4}{3} = \frac{4}{3}$ The solution is $\frac{4}{3}$.	9. Solve: $\frac{3}{2} \cdot t = \frac{7}{8}$. A. $\frac{21}{16}$ B. 1 C. $\frac{7}{12}$ D. $\frac{7}{24}$

Objective [2.3a] Add using fractional notation when denominators are the same.		
Brief Procedure	**Example**	**Practice Exercise**
Add the numerators, keep the denominator, and simplify, if possible.	Add and simplify: $\frac{3}{8}+\frac{7}{8}$. $\frac{3}{8}+\frac{7}{8}=\frac{3+7}{8}=\frac{10}{8}=\frac{2\cdot 5}{2\cdot 4}=\frac{2}{2}\cdot\frac{5}{4}=$ $1\cdot\frac{5}{4}=\frac{5}{4}$	10. Add and simplify: $\frac{1}{12}+\frac{7}{12}$ A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{7}{144}$

Objective [2.3b] Add using fractional notation when denominators are different, by multiplying by 1 to find the least common denominator.		
Brief Procedure	**Example**	**Practice Exercise**
a) Find the least common multiple of the denominators. That number is the least common denominator, LCD. b) Multiply by 1, using an appropriate notation, n/n, to express each number in terms of the LCD. c) Add the numerators, keeping the same denominator. d) Simplify,if possible.	Add and simplify, if possible: $\frac{2}{9}+\frac{1}{6}$. $9=3\cdot 3$ and $6=2\cdot 3$ so the LCM of 9 and 6 is $2\cdot 3\cdot 3$, or 18. Thus the LCD is 18. $\frac{2}{9}+\frac{1}{6}$ $=\frac{2}{9}\cdot\frac{2}{2}+\frac{1}{6}\cdot\frac{3}{3}$ $=\frac{4}{18}+\frac{3}{18}$ $=\frac{7}{18}$ No simplification is necessary.	11. Add and simplify, if possible: $\frac{3}{4}+\frac{3}{10}$. A. $\frac{3}{7}$ B. $\frac{3}{14}$ C. $\frac{21}{20}$ D. $\frac{9}{40}$

Objective [2.3c] Subtract using fractional notation.

Brief Procedure	Example	Practice Exercise
If denominators are the same, subtract the numerators, keep the denominator, and simplify, if possible. If denominators are different, a) Find the least common multiple of the denominators. That number is the least common denominator, LCD. b) Multiply by 1, using an appropriate notation, n/n, to express each number in terms of the LCD. c) Subtract the numerators, keeping the same denominator. d) Simplify, if possible.	Subtract and simplify, if possible: $\frac{2}{3} - \frac{1}{4}$. The LCM of 3 and 4 is 12, so the LCD is 12. $\frac{2}{3} - \frac{1}{4} = \frac{2}{3} \cdot \frac{4}{4} - \frac{1}{4} \cdot \frac{3}{3}$ $= \frac{8}{12} - \frac{3}{12}$ $= \frac{5}{12}$ No simplification is necessary.	12. Subtract and simplify, if possible: $\frac{4}{5} - \frac{3}{8}$ A. $\frac{17}{40}$ B. $\frac{7}{40}$ C. $\frac{3}{10}$ D. $\frac{1}{3}$

Objective [2.3d] Use $<$ or $>$ with fractional notation to write a true sentence.

Brief Procedure	Example	Practice Exercise
Multiply by 1 to make the denominators the same, if necessary. Then compare the numerators. The fraction with the larger numerator is the larger fraction.	Use $<$ or $>$ for $\square$ to write a true sentence: $\frac{3}{5} \square \frac{5}{8}$. $\frac{3}{5} \cdot \frac{8}{8} = \frac{24}{40}$ $\frac{5}{8} \cdot \frac{5}{5} = \frac{25}{40}$ Since $24 < 25$, it follows that $\frac{3}{5} < \frac{5}{8}$.	13. Use $<$ or $>$ for $\square$ to write a true sentence: $\frac{2}{3} \square \frac{5}{9}$. A. $<$ B. $>$

Objective [2.3e] Solve equations of the type $x + a = b$ and $a + x = b$, where a and b may be fractions.

Brief Procedure	Example	Practice Exercise
Subtract a on both sides of the equation.	Solve: $x + \frac{1}{3} = \frac{4}{5}$. $x + \frac{1}{3} = \frac{4}{5}$ $x + \frac{1}{3} - \frac{1}{3} = \frac{4}{5} - \frac{1}{3}$ $x + 0 = \frac{4}{5} \cdot \frac{3}{3} - \frac{1}{3} \cdot \frac{5}{5}$ $x = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$	14. Solve: $x + \frac{5}{6} = \frac{7}{8}$. A. $\frac{1}{24}$ B. $\frac{41}{24}$ C. $\frac{41}{48}$ D. 1

Objective [2.4a] Convert from mixed numerals to fractional notation and from fractional notation to mixed numerals.

Brief Procedure	Example	Practice Exercise
To convert from mixed numerals to fractional notation: a) Multiply the whole number by the denominator. b) Add the result to the numerator. c) Keep the denominator.	Convert $5\frac{3}{8}$ to fractional notation. $5 \times 8 = 40$ and $40 + 3 = 43$, so $5\frac{3}{8} = \frac{43}{8}$.	15. Convert $3\frac{4}{5}$ to fractional notation. A. $\frac{7}{5}$ B. $\frac{12}{5}$ C. $\frac{19}{5}$ D. $\frac{34}{5}$
To convert from fractional notion to mixed numerals, divide the numerator by the denominator. The quotient is the whole number part of the mixed numeral. The numerator of the fractional part is the remainder, and the denominator is the denominator of the fractional notation.	Convert $\frac{13}{3}$ to a mixed numeral. $\begin{array}{r} 4 \\ 3\overline{)13} \\ 12 \\ \hline 1 \end{array}$ $\quad \frac{13}{3} = 4\frac{1}{3}$	16. Convert $\frac{11}{6}$ to a mixed numeral. A. $1\frac{1}{6}$ B. $1\frac{5}{6}$ C. $2\frac{1}{6}$ D. $2\frac{5}{6}$

Objective [2.4b] Add using mixed numerals.

Brief Procedure	Example	Practice Exercise
First add the fractions. Then add the whole numbers.	Add: $3\frac{5}{8} + 4\frac{1}{2}$. The LCD is 8. $\begin{array}{rcl} 3\,\frac{5}{8} & = & 3\,\frac{5}{8} \\ +4\,\frac{1}{2}\cdot\frac{4}{4} & = & +4\,\frac{4}{8} \\ \hline & & 7\,\frac{9}{8} = 7 + \frac{9}{8} \\ & & = 7 + 1\frac{1}{8} \\ & & = 8\frac{1}{8} \end{array}$	17. Add: $5\frac{2}{3} + 1\frac{3}{4}$. A. $7\frac{5}{12}$ B. $6\frac{5}{7}$ C. $6\frac{5}{12}$ D. $6\frac{1}{2}$

Objective [2.4c] Subtract using mixed numerals.		
Brief Procedure	**Example**	**Practice Exercise**
First subtract the fractions, borrowing if necessary. Then subtract the whole numbers.	Subtract: $6\frac{1}{3} - 4\frac{1}{2}$. $\begin{array}{rcrcr} 6\,\frac{1}{3}\cdot\frac{2}{2} & = & 6\,\frac{2}{6} & = & 5\,\frac{8}{6} \\ -4\,\frac{1}{2}\cdot\frac{3}{3} & = & -4\,\frac{3}{6} & = & -4\,\frac{3}{6} \\ \hline & & & & 1\,\frac{5}{6} \end{array}$	18. Subtract: $9\frac{3}{8} - 3\frac{3}{4}$. A. $5\frac{3}{8}$ B. $5\frac{5}{8}$ C. $6\frac{3}{8}$ D. $6\frac{5}{8}$

Objective [2.4d] Multiply using mixed numerals.		
Brief Procedure	**Example**	**Practice Exercise**
First convert to fractional notation and multiply. Then convert the result to a mixed numeral, if appropriate.	Multiply: $1\frac{3}{8} \cdot 5\frac{2}{3}$. $1\frac{3}{8} \cdot 5\frac{2}{3} = \frac{11}{8} \cdot \frac{17}{3} = \frac{187}{24} = 7\frac{19}{24}$	19. Multiply: $6\frac{2}{5} \cdot 2\frac{3}{4}$. A. $12\frac{3}{10}$ B. $14\frac{1}{5}$ C. $15\frac{9}{10}$ D. $17\frac{3}{5}$

Objective [2.4e] Divide using mixed numerals.		
Brief Procedure	**Example**	**Practice Exercise**
First convert to fractional notation and divide. Then convert the result to a mixed numeral, if appropriate.	Divide: $4\frac{2}{3} \div 2\frac{3}{5}$. $4\frac{2}{3} \div 2\frac{3}{5} = \frac{14}{3} \div \frac{13}{5} = \frac{14}{3} \cdot \frac{5}{13} =$ $\frac{70}{39} = 1\frac{31}{39}$	20. Divide: $9\frac{1}{8} \div 3\frac{1}{4}$. A. $2\frac{5}{8}$ B. $2\frac{21}{26}$ C. $3\frac{5}{26}$ D. $3\frac{1}{2}$

Objective [2.5a] Solve applied problems involving addition, subtraction, multiplication, and division using fractional notation and mixed numerals.

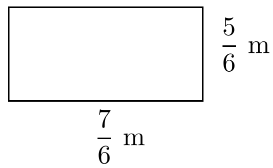
Brief Procedure

Use the five-step problem solving process.

Example

A rectangular rug is $\frac{7}{6}$ m long and $\frac{5}{6}$ m wide. What is its area?.

1. *Familiarize.* We make a drawing.



Recall that the area of a rectangle is length times width. Let $A =$ the area of the rug.

2. *Translate.*

$$\begin{array}{ccccc} \underbrace{\text{Area}} & \text{is} & \underbrace{\text{length}} & \text{times} & \underbrace{\text{width}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ A & = & \frac{7}{6} & \times & \frac{5}{6} \end{array}$$

3. *Solve.* We multiply.

$$A = \frac{7}{6} \times \frac{5}{6} = \frac{7 \times 5}{6 \times 6} = \frac{35}{36}$$

4. *Check.* We can repeat the calculation. The answer checks.
5. *State.* The area of the rug is $\frac{35}{36}$ m^2.

Practice Exercise

21. Sam is $73\frac{1}{4}$ in. tall, and Ray is $70\frac{1}{2}$ in. tall. How much taller is Sam?

A. $3\frac{3}{4}$ in.

B. $3\frac{1}{4}$ in.

C. $2\frac{3}{4}$ in.

D. $2\frac{1}{4}$ in.

Objective [2.6a] Simplify expressions using the rules for order of operations.

Brief Procedure

1. Do all calculations within parentheses before operations outside.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

Example

Simplify: $\left(\frac{3}{2}\right)^2 \div 9 + \frac{4}{5} \cdot \frac{1}{3}$.

$$\begin{aligned} &\left(\frac{3}{2}\right)^2 \div 9 + \frac{4}{5} \cdot \frac{1}{3} \\ &= \frac{9}{4} \div 9 + \frac{4}{5} \cdot \frac{1}{3} \\ &= \frac{9}{4} \cdot \frac{1}{9} + \frac{4}{5} \cdot \frac{1}{3} \\ &= \frac{\cancel{9} \cdot 1}{4 \cdot \cancel{9}} + \frac{4 \cdot 1}{5 \cdot 3} \\ &= \frac{1}{4} + \frac{4}{15} \\ &= \frac{15}{60} + \frac{16}{60} \\ &= \frac{31}{60} \end{aligned}$$

Practice Exercise

22. Simplify: $3\left(\frac{2}{3} - \frac{1}{6}\right) - \left(\frac{1}{4}\right)^2 \cdot \frac{8}{9}$.

A. $\frac{5}{9}$

B. $\frac{8}{9}$

C. $\frac{13}{9}$

D. $\frac{16}{9}$

Objective [2.6b] Estimate with fractional and mixed numeral notation.		
Brief Procedure	Example	Practice Exercise
Estimate a fraction (or the fractional part of a mixed numeral) as 0 when the numerator is very small in comparison to the denominator. Estimate a fraction as $\frac{1}{2}$ when the denominator is about twice the numerator. Estimate a fraction as 1 when the numerator is nearly equal to the denominator.	Estimate as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$: $\frac{3}{4} + 2\frac{1}{8} - \frac{12}{25}$. $\frac{3}{4} + 2\frac{1}{8} - \frac{12}{25} \approx 1 + 2 - \frac{1}{2} = 2\frac{1}{2}$, or $\frac{5}{2}$	23. Estimate as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$: $3\frac{5}{9} - 1\frac{1}{12} + \frac{19}{23}$. A. $2\frac{1}{2}$ B. 3 C. $3\frac{1}{2}$ D. 4