Developmental Mathematics
Chapter 16 Review

Objective [16.1a] Write a quadratic equation in standard form $ax^2 + bx + c = 0$, $a > 0$, and determine the coefficients a, b, and c.

Brief Procedure	Example	Practice Exercise
Given an quadratic equation, use the addition and multiplication principles to write an equivalent equation in standard form, $ax^2 + bx + c = 0$, $a > 0$.	Write $-2x^2 + 3x = 5$ in standard form and determine a, b, and c. First we subtract 5 on both sides of the equation. Then we multiply by -1 on both sides. $-2x^2 + 3x = 5$ $-2x^2 + 3x - 5 = 0$ $2x^2 - 3x + 5 = 0$ With the equation in standard form, we see that $a = 2$, $b = -3$, and $c = 5$.	1. Write $x^2 + 6 = 4x$ in standard form and determine a, b, and c. Which of the following is true? A. $b = -4$ B. $b = 1$ C. $b = 4$ D. $b = 6$

Objective [16.1b] Solve quadratic equations of the type $ax^2 + bx = 0$, where $b \neq 0$, by factoring.

Brief Procedure	Example	Practice Exercise
Factor $ax^2 + bx$ and then use the principle of zero products. An equation of the type $ax^2 + bx = 0$ will always have 0 as one solution and a nonzero number as the other solution.	Solve: $5x^2 - 4x = 0$. $5x^2 - 4x = 0$ $x(5x - 4) = 0$ $x = 0$ *or* $5x - 4 = 0$ $x = 0$ *or* $5x = 4$ $x = 0$ *or* $x = \frac{4}{5}$ The solutions are 0 and $\frac{4}{5}$.	2. Solve: $2x^2 + 3x = 0$. A. $0, -3$ B. $0, -\frac{3}{2}$ C. $0, -\frac{2}{3}$ D. $0, \frac{3}{2}$

Objective [16.1c] Solve quadratic equations of the type $ax^2 + bx + c = 0$, where $b \neq 0$ and $c \neq 0$, by factoring.

Brief Procedure	Example	Practice Exercise
Factor $ax^2 + bx + c$ and then use the principle of zero products.	Solve: $2x^2 + 5x = 3$. First we write the equation in standard form. Then we factor and use the principle of zero products. $2x^2 + 5x = 3$ $2x^2 + 5x - 3 = 0$ $(2x - 1)(x + 3) = 0$ $2x - 1 = 0$ *or* $x + 3 = 0$ $2x = 1$ *or* $x = -3$ $x = \frac{1}{2}$ *or* $x = -3$ The solutions are $\frac{1}{2}$ and -3.	3. Solve: $x^2 + 20 = 9x$. A. One solution is -5. B. One solution is -4. C. One solution is 3. D. One solution is 4.

<table>
<tr><td colspan="3">Objective [16.1d] Solve applied problems involving quadratic equations.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Use the five-step problem solving process.</td>
<td>The number of diagonals d of a polygon of n sides is given by the formula
$d = \dfrac{n^2 - 3n}{2}$.
If a polygon has 5 diagonals, how many sides does it have?
1. Familiarize. We will use the formula given above.
2. Translate. We substitute 5 for d in the formula.
$5 = \dfrac{n^2 - 3n}{2}$
3. Solve. We solve the equation for n, first reversing the equation for convenience.
$\dfrac{n^2 - 3n}{2} = 5$
$n^2 - 3n = 10$
$n^2 - 3n - 10 = 0$
$(n-5)(n+2) = 0$
$n - 5 = 0 \quad or \quad n + 2 = 0$
$n = 5 \quad or \quad n = -2$
4. Check. Since the number of sides cannot be negative, -2 cannot be a solution. We substitute 5 for n in the formula to verify that it is a solution.
$d = \dfrac{n^2 - 3n}{2} = \dfrac{5^2 - 3 \cdot 5}{2}$
$= \dfrac{25 - 15}{2} = \dfrac{10}{2}$
$= 5$
Since $d = 5$ when $n = 5$, the number 5 checks.
5. State. The polygon has 5 sides.</td>
<td>4. If a polygon has 9 diagonals, how many sides does it have?
A. 5
B. 6
C. 7
D. 8</td>
</tr>
</table>

Objective [16.2a] Solve quadratic equations of the type $ax^2 = p$.		
Brief Procedure	Example	Practice Exercise
Solve for x^2 and then use the principle of square roots: a) The equation $x^2 = d$ has two real solutions when $d > 0$. The solutions are $\sqrt{d}$ and $-\sqrt{d}$. b) The equation $x^2 = 0$ has 0 as its only solution. c) The equation $x^2 = d$ has no real-number solution when $d < 0$.	Solve: $3x^2 = 15$. $3x^2 = 15$ $x^2 = 5$ $x = \sqrt{5}$ *or* $x = -\sqrt{5}$ The solutions are $\sqrt{5}$ and $-\sqrt{5}$.	5. Solve: $3x^2 - 2 = 0$. A. $\sqrt{2}, -\sqrt{2}$ B. $\sqrt{3}, -\sqrt{3}$ C. $\frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}$ D. $\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}$

Objective [16.2b] Solve quadratic equations of the type $(x + c)^2 = d$.		
Brief Procedure	Example	Practice Exercise
Use the principle of square roots.	Solve: $x^2 - 2x + 1 = 25$. $x^2 - 2x + 1 = 25$ $(x - 1)^2 = 25$ $x - 1 = 5$ *or* $x - 1 = -5$ $x = 6$ *or* $x = -4$ The solutions are 6 and -4.	6. Solve: $(x + 3)^2 = 7$. A. $-7 \pm \sqrt{3}$ B. $7 \pm \sqrt{3}$ C. $-3 \pm \sqrt{7}$ D. $3 \pm \sqrt{7}$

<table>
<tr><td colspan="3">Objective [16.2c] Solve quadratic equations by completing the square.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr><td>1. If $a \neq 1$, multiply by $1/a$ so that the x^2-coefficient is 1.
2. If the x^2-coefficient is 1, add so that the equation is in the form $x^2 + bx = -c$, or $x^2 + \frac{b}{a}x = -\frac{c}{a}$ if step (1) has been applied.
3. Take half of the x-coefficient and square it. Add the result on both sides of the equation.
4. Express the side with the variables as the square of a binomial.
5. Use the principle of square roots and complete the solution.</td><td>Solve: $2x^2+2x-3=0$ by completing the square.

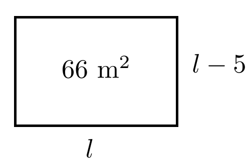
First, we multiply by $\frac{1}{2}$ on both sides of the equation to make the x^2-coefficient 1.
$$\begin{aligned} 2x^2+2x-3 &= 0 \\ \frac{1}{2}(2x^2+2x-3) &= \frac{1}{2}\cdot 0 \\ x^2+x-\frac{3}{2} &= 0 \\ x^2+x &= \frac{3}{2} \end{aligned}$$
Now we add $\left(\frac{b}{2}\right)^2$, or $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$ on both sides.
$$x^2+x+\frac{1}{4} = \frac{3}{2}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2 = \frac{7}{4}$$
$x+\frac{1}{2}=\frac{\sqrt{7}}{2}$ *or* $x+\frac{1}{2}=-\frac{\sqrt{7}}{2}$
$x=-\frac{1}{2}+\frac{\sqrt{7}}{2}$ *or* $x=-\frac{1}{2}-\frac{\sqrt{7}}{2}$
The solutions are $\frac{-1\pm\sqrt{7}}{2}$.</td><td>7. Solve: $x^2+2x-5=0$.
A. $1, -3$
B. $-1\pm\sqrt{5}$
C. $-1\pm\sqrt{6}$
D. $-1\pm\sqrt{7}$</td></tr>
</table>

Objective [16.2d] Solve certain problems involving quadratic equations of the type $ax^2 = p$.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	An object is dropped from the top of a 1214-m high building. How long will it take the object to reach the ground? 1. *Familiarize.* A formula that fits this situation is $s = 16t^2$, where s is the distance, in feet, traveled by a body falling freely from rest in t seconds. Here we know that s is 1214 m and we want to find t. 2. *Translate.* We substitute 1214 for s in the formula. $1214 = 16t^2$ 3. *Solve.* $1214 = 16t^2$ $\frac{1214}{16} = t^2$ $75.875 = t^2$ $\sqrt{75.875} = t \quad or \quad -\sqrt{78.875} = t$ $8.7 \approx t \quad or \quad -8.7 \approx t$ 4. *Check.* Time cannot be negative in this situation, so -8.7 cannot be a solution. We substitute 8.7 for t in the formula: $s = 16(8.7)^2 = 16(75.69) = 1211.04$ Note that $1211.04 \approx 1214$; since we approximated the solution, we have a check. 5. *State.* It would take the object about 8.7 sec to reach the ground.	8. The Chrysler Building in New York is 1046 ft tall. How long would it take an object to fall to the ground from the top? A. About 7.9 sec B. About 8.1 sec C. About 8.2 sec D. About 8.5 sec

Objective [16.3a] Solve quadratic equations using the quadratic formula.		
Brief Procedure	Example	Practice Exercise
The solutions of the equation $ax^2+bx+c=0$ are given by the formula $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$	Solve using the quadratic formula: $x^2+4x=3$. First we find standard form and determine a, b, and c. $x^2+4x-3=0$ $a=1, b=4, c=-3$ Then we use the quadratic formula. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-4\pm\sqrt{4^2-4\cdot 1\cdot(-3)}}{2\cdot 1}$ $x=\frac{-4\pm\sqrt{16+12}}{2}=\frac{-4\pm\sqrt{28}}{2}$ $x=\frac{-4\pm\sqrt{4\cdot 7}}{2}=\frac{-4\pm\sqrt{4}\sqrt{7}}{2}$ $x=\frac{-4\pm 2\sqrt{7}}{2}=\frac{2(-2\pm\sqrt{7})}{2\cdot 1}$ $x=\frac{2}{2}\cdot\frac{-2\pm\sqrt{7}}{1}=-2\pm\sqrt{7}$ The solutions are $-2+\sqrt{7}$ and $-2-\sqrt{7}$, or $-2\pm\sqrt{7}$.	9. Solve $2x^2-3x-7=0$ using the quadratic formula. A. $\frac{-3\pm\sqrt{65}}{4}$ B. $\frac{-3\pm\sqrt{65}}{2}$ C. $\frac{3\pm\sqrt{65}}{4}$ D. $\frac{3\pm\sqrt{65}}{2}$

Objective [16.3b] Find approximate solutions of quadratic equations using a calculator.		
Brief Procedure	Example	Practice Exercise
Use a calculator to find the approximate value of solutions found using the quadratic formula.	Use a calculator to approximate the solutions of $x^2+4x=3$ to the nearest tenth. In Objective 16.3a we used the quadratic formula to find that the solutions of this equation are $-2\pm\sqrt{7}$. Using a calculator and rounding to the nearest tenth, we have $-2+\sqrt{7}\approx 0.6457513111\approx 0.6$ and $-2-\sqrt{7}\approx -4.645751311\approx -4.6$. The approximate solutions are 0.6 and -4.6.	10. Use a calculator to approximate the solutions of $x^2-5x+2=0$ to the nearest tenth. A. $3.4, -0.4$ B. $4.6, 0.4$ C. $5.4, -0.4$ D. $5.8, 0.4$

Objective [16.4a] Solve a formula for a given letter.		
Brief Procedure	Example	Practice Exercise
Use an appropriate equation-solving technique to get the letter alone on one side of the equation.	Solve $m^2 + n^2 = r^2$ for n. $m^2 + n^2 = r^2$ $n^2 = r^2 - m^2$ $n = \sqrt{r^2 - m^2}$	11. Solve $A = cd^2$ for d. A. $d = \frac{A}{c}$ B. $d = \sqrt{Ac}$ C. $d = \frac{c}{A}$ D. $d = \sqrt{\frac{A}{c}}$

Objective [16.5a] Solve applied problems using quadratic equations.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	The width of a rectangle is 5 m less than the length. The area is 66 m^2. Find the length and the width. 1. *Familiarize.* We first make a drawing. Let l = the length. Then $l - 5$ = the width.  2. *Translate.* Recall that the area of a rectangle is length × width. Thus, we have $l(l-5) = 66.$ 3. *Solve.* $l(l-5) = 66$ $l^2 - 5l = 66$ $l^2 - 5l - 66 = 0$ $(l-11)(l+6) = 0$ $l - 11 = 0$ *or* $l + 6 = 0$ $l = 11$ *or* $l = -6$ 4. *Check.* Length cannot be negative, so -6 is not a solution. If $l = 11$, then $l - 5 = 11 - 5 = 6$ and the area is $11 \cdot 6$, or 66 m^2. This checks. 5. *State.* The length is 11 m and the width is 6 m.	12. The speed of a boat in still water is 10 km/h. The boat travels 24 km upstream and 24 km downstream in a total time of 5 hr. What is the speed of the stream? A. 2 km/h B. 3 km/h C. 4 km/h D. 6 km/h

Objective [16.6a] Graph quadratic equations.

Brief Procedure

The graph of a quadratic equation $y = ax^2 + bx + c$ is a parabola. It is convenient to find the vertex first. The x-coordinate is $-\dfrac{b}{2a}$. The y-coordinate of the vertex is found by substituting the x-coordinate into the equation and computing y. The line of symmetry is $x = -\dfrac{b}{2a}$. After finding the vertex and line of symmetry, choose some x-values on either side of the vertex and find the corresponding y-values. Then plot these points, along with the vertex, and draw the graph.

Example

Graph: $y = x^2 - 2x - 3$.

First we find the vertex. The x-coordinate is

$$-\frac{b}{2a} = -\frac{-2}{2 \cdot 1} = 1.$$

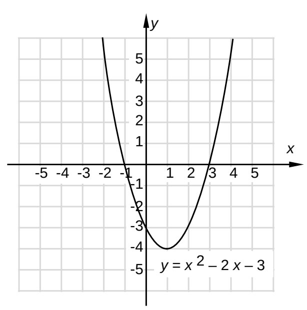
We substitute 1 for x into the equation to find the y-coordinate of the vertex.

$$\begin{aligned} y &= x^2 - 2x - 3 \\ &= 1^2 - 2 \cdot 1 - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

The vertex is $(1, -4)$. The line of symmetry is $x = 1$. Now we choose some x-values on either side of the vertex, find the corresponding y-values, plot points, and graph the parabola.

For $x = -1, y = (-1)^2 - 2(-1) - 3 = 0$.
For $x = 0, y = 0^2 - 2 \cdot 0 - 3 = -3$.
For $x = 2, y = 2^2 - 2 \cdot 2 - 3 = -3$.
For $x = 3, y = 3^2 - 2 \cdot 3 - 3 = 0$.

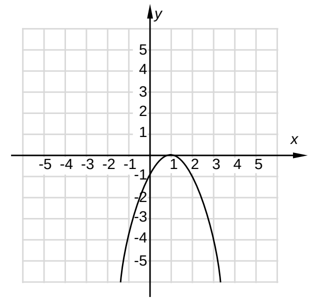
x	y	
1	-4	←Vertex
-1	0	
0	-3	
2	-3	
3	0	



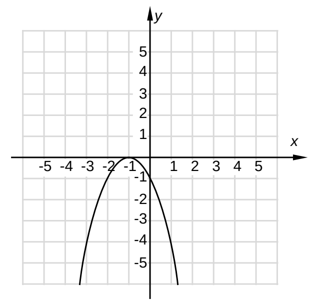
Practice Exercise

13. Graph: $y = x^2 + 2x + 1$.

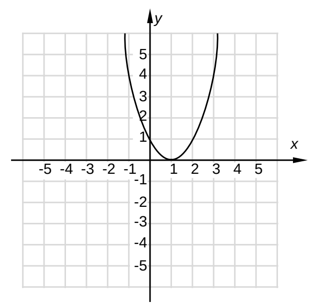
A.



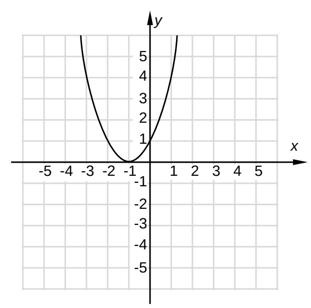
B.



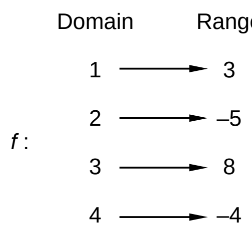
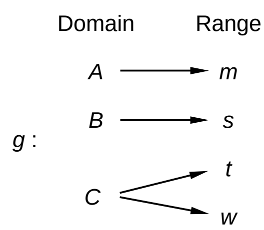
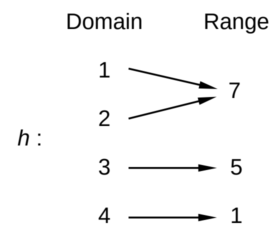
C.



D.



Objective [16.7a] Determine whether a correspondence is a function.

Brief Procedure	Example	Practice Exercise
A function is a correspondence between a first set, called the domain, and a second set, called the range, such that each member of the domain corresponds to exactly one member of the range.	Determine whether each correspondence is a function. a)  b)  a) f is a function because each member of the domain corresponds to exactly one member of the range. b) g is not a function because one member of the domain, C, corresponds to more than one member of the range.	14. Determine whether the correspondence is a function.  A. Yes B. No

Objective [16.7b] Given a function described by an equation, find function values (outputs) for specified values (inputs).

Brief Procedure	Example	Practice Exercise
Evaluate the function for the value of the given input.	Find $f(-1)$ for $f(x) = 2x^2 - 1$. $f(-1) = 2(-1)^2 - 1 = 2 - 1 = 1.$	15. Find $g(2)$ for $g(x) = 3x - 5$. A. -11 B. -2 C. 1 D. 8

Objective [16.7c] Draw the graph of a function.

Brief Procedure

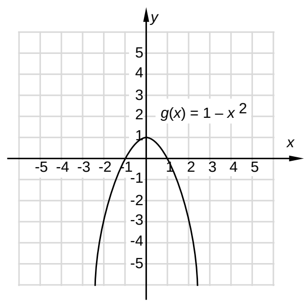
Find some function values, plot points, and draw the graph.

Example

Graph: $g(x) = 1 - x^2$.

We find some function values, plot points, and draw the graph.

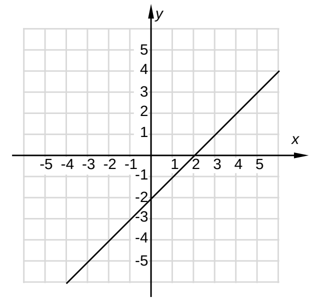
x	$g(x)$
-2	-3
-1	0
0	1
1	0
2	-3



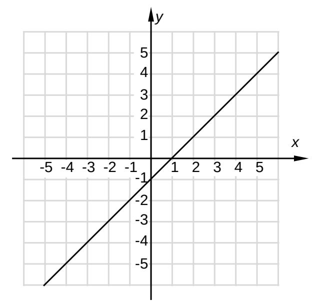
Practice Exercise

16. Graph: $f(x) = x - 1$.

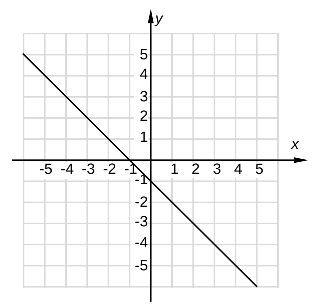
A.



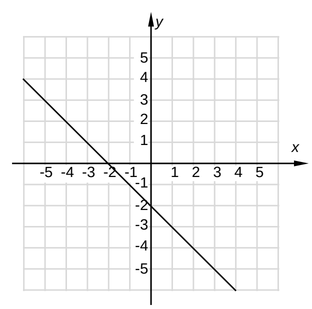
B.



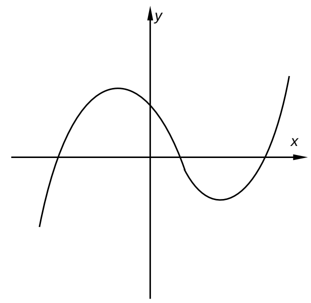
C.

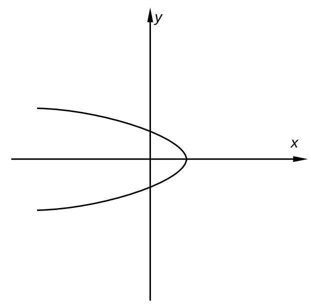


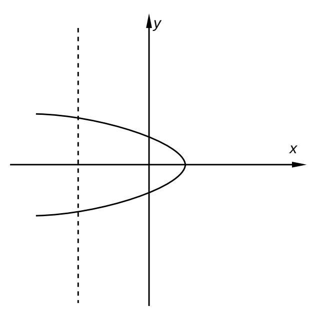
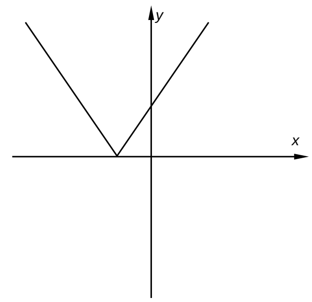
D.



<table>
<tr><td colspan="3">Objective [16.7d] Determine whether a graph is that of a function.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>A graph represents a function if it is impossible to draw a vertical line that intersects the graph more than once. This is the vertical-line test.</td>
<td>Determine whether each is the graph of a function.

a)


b)


a) The graph is that of a function because no vertical line can cross the graph at more than one point. This can be confirmed with a straight edge.
b) The graph is not that of a function because a vertical line can be drawn that crosses the graph more than once.

</td>
<td>17. Determine whether the graph is the graph of a function.


A. Yes
B. No</td>
</tr>
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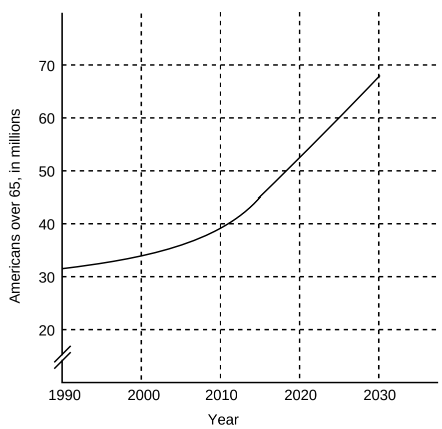
<table>
<tr><td colspan="3">Objective [16.7e] Solve applied problems involving functions and their graphs.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Read data from the graph.</td>
<td>The graph below shows the number of Americans over age 65 as a function of the year. (The data is projected for 2000-2030.)


Use the graph to approximate the number of Americans over age 65 in 2000.

Locate 2000 on the horizontal axis, move directly up to the graph, and then across to the vertical axis. We see that the output that corresponds to the input 2000 is about 34, so there will be about 34 million Americans over age 65 in 2000.</td>
<td>18. Use the graph at the left to determine the year in which there will be about 52 million Americans over 65.

A. 2000
B. 2010
C. 2020
D. 2030</td>
</tr>
</table>