Developmental Mathemtics
Chapter 13 Review

Objective [13.1a] Given the coordinates of two points on a line, find the slope of the line.

Brief Procedure	Example	Practice Exercise
The slope of a line containing points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\text{rise}}{\text{run}} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$	Find the slope, if it exists, of the line containing the points $(-1, 5)$ and $(2, -3)$. Consider (x_1, y_1) to be $(-1, 5)$ and (x_2, y_2) to be $(2, -3)$. $\text{Slope} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{2-(-1)} = \frac{-8}{3}, \text{ or } -\frac{8}{3}$ Note that we would have gotten the same result if we had considered (x_1, y_1) to be $(2, -3)$ and (x_2, y_2) to be $(-1, 5)$. We can subtract in either order as long as the x-coordinates are subtracted in the same order in which the y-coordinates are subtracted.	1. Find the slope, if it exists, of the line containing the points $(6, -2)$ and $(8, -1)$. A. -2 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 2

Objective [13.1b] Find the slope of a line from an equation.

Brief Procedure	Example	Practice Exercise
To find the slope of a non-vertical line given in an equation $Ax + By = C$, solve the equation for y and get the resulting equation in the form $y = mx + b$. The coefficient of the x-term, m, is the slope of the line. The slope of a vertical line is undefined.	Find the slope, if it exists, of each line. a) $3x + 4y = 8$ b) $y = -1$ c) $x = 2$ a) We solve for y to get the equation in the form $y = mx + b$. $3x + 4y = 8$ $4y = -3x + 8$ $y = \frac{-3x+8}{4}$ $y = -\frac{3}{4}x + 2$ The slope is $-\frac{3}{4}$. b) We can think of $y = -1$ as $y = 0x - 1$. Then we see that the slope is 0. Note that the graph of this equation is a horizontal line. The slope of any horizontal line is 0. c) The graph of $x = 2$ is a vertical line, so the slope is undefined.	2. Find the slope, if it exists, of the line $2x - 3y = 12$. A. $\frac{3}{2}$ B. $\frac{2}{3}$ C. $-\frac{2}{3}$ D. -4

Objective [13.1c] Find the slope or rate of change in an applied problem involving slope.		
Brief Procedure	Example	Practice Exercise
Determine the rise and run, or the change in y and the change in x, and compute the slope, or rate of change.	A road rises 40 m over a horizontal distance of 1250 m. Find the grade of the road. $\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{40}{1250} = 0.032 = 3.2\%$	3. A set of stairs rises 12 ft over a horizontal distance of 150 ft. Find the grade of the stairs. A. 8% B. 12% C. 12.5% D. 15%

Objective [13.2a] Given an equation in the form $y = mx + b$, find the slope and the y-intercept; and find an equation of a line when the slope and the y-intercept are given.		
Brief Procedure	Example	Practice Exercises
In the equation $y = mx + b$, the slope is m and the y-intercept is $(0, b)$.	Find the slope and y-intercept of $3x + 5y = 15$. We solve the equation for y: $3x + 5y = 15$ $5y = -3x + 15$ $\frac{5y}{5} = \frac{-3x + 15}{5}$ $y = \frac{-3x}{5} + \frac{15}{5}$ $y = -\frac{3}{5}x + 3$ Now that the equation is in the form $y = mx + b$, we see that the slope is $-\frac{3}{5}$ and the y-intercept is (0,3).	4. For the graph of $4x - 3y = 12$, which of the following is true? A. The slope is $\frac{3}{4}$. B. The slope is $-\frac{3}{4}$. C. The y-intercept is $(0, -4)$. D. The y-intercept is $(0, 4)$.
When the slope m and the y-intercept $(0, b)$ of a line are given, find an equation of the line by substituting in the equation $y = mx + b$.	A line has slope -3 and y-intercept (0,2). Find an equation of the line. We substitute -3 for m and 2 for b in the slope-intercept equation. $y = mx + b$ $y = -3x + 2$	5. A line has slope 4 and y-intercept $(0, -1)$. Find an equation of the line. A. $y = -x + 4$ B. $y = -x - 4$ C. $y = 4x - 1$ D. $y = 4x + 1$

Objective [13.2b] Find an equation of a line when the slope and a point on the line are given.

Brief Procedure	Example	Practice Exercise
Substitute the given slope for m in the slope-intercept equation $y = mx+b$ and then substitute the coordinates of the given point to find b.	Find an equation of the line with slope -2 that contains the point $(3, -1)$. We know that the slope is -2, so the equation is $y = -2x + b$. Using the point $(3, -1)$, we substitute 3 for x and -1 for y in $y = -2x + b$. $y = -2x + b$ $-1 = -2 \cdot 3 + b$ $-1 = -6 + b$ $5 = b$ Then the equation is $y = -2x + 5$.	6. Find an equation of the line with slope 4 that contains the point $(-2, -5)$. A. $y = 4x - 5$ B. $y = 4x + 18$ C. $y = 4x - 2$ D. $y = 4x + 3$

Objective [13.2c] Find an equation of a line when two points on the line are given.

Brief Procedure	Example	Practice Exercise
Use the two given points to find the slope of the line. Next, substitute the slope for m in the slope-intercept equation $y = mx+b$ and then substitute the coordinates of either of the given points to find b.	Find an equation of the line containing the points $(4, 3)$ and $(-2, 5)$. First, we find the slope. $m = \frac{3-5}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$ Thus, $y = -\frac{1}{3}x + b$. Now use either of the given points to find b. We use (4,3) and substitute 4 for x and 3 for y. $y = -\frac{1}{3}x + b$ $3 = -\frac{1}{3} \cdot 4 + b$ $3 = -\frac{4}{3} + b$ $\frac{13}{3} = b$ Then the equation of the line is $y = -\frac{1}{3}x + \frac{13}{3}$.	7. Find an equation of the line containing $(-3, -2)$ and (3,4). A. $y = x + 1$ B. $y = x - 7$ C. $y = -x + 1$ D. $y = -x - 7$

Objective [13.3a] Determine whether the graphs of two linear equations are parallel.

Brief Procedure	Example	Practice Exercise
Parallel nonvertical lines have the same slope and different y-intercepts. Parallel vertical lines have equations $x = p$ and $x = q$, where $p \neq q$.	Determine whether the graphs of the lines $y = -2x + 1$ and $4x + 2y = 5$ are parallel. The first equation is in slope-intercept form ($y = mx + b$), so we see that it has slope -2 and y-intercept (0,1). We solve the second equation for y. $4x + 2y = 5$ $2y = -4x + 5$ $y = \frac{1}{2}(-4x + 5)$ $y = -2x + \frac{5}{2}$ Thus, the slope of the second line is -2 and its y-intercept is $\left(0, \frac{5}{2}\right)$. Since the two lines have the same slope, -2, and different y-intercepts, (0,1) and $\left(0, \frac{5}{2}\right)$, they are parallel.	8. Determine whether the graphs of the lines $x + y = 3$ and $x - y = 3$ are parallel. A. Yes B. No

Objective [13.3b] Determine whether the graphs of two linear equations are perpendicular.

Brief Procedure	Example	Practice Exercise
Two nonvertical lines are perpendicular if the product of their slopes is -1. If one equation in a pair of perpendicular lines is vertical, then the other is horizontal. That is, two lines with equations $x = a$ and $y = b$ are perpendicular.	Determine whether the graphs of the lines $2x + y = 4$ and $x + 2y = 3$ are perpendicular. We first solve each equation for y in order to determine the slopes. a) $2x + y = 4$ $y = -2x + 4$ b) $x + 2y = 3$ $2y = -x + 3$ $y = \frac{1}{2}(-x + 3)$ $y = -\frac{1}{2}x + \frac{3}{2}$ The slopes are -2 and $-\frac{1}{2}$. The product of the slopes is $-2\left(-\frac{1}{2}\right) = 1$. Since the product of the slopes is not -1, the lines are not perpendicular.	9. Determine whether the graphs of the lines $3x - 2y = 4$ and $4x + 6y = 3$ are perpendicular. A. Yes B. No

Objective [13.4a] Determine whether an ordered pair of numbers is a solution of an inequality in two variables.

Brief Procedure	Example	Practice Exercise
Following alphabetical order, substitute the coordinates of the ordered pair in the inequality and determine whether a true inequality results.	Determine whether $(4, -1)$ is a solution of $x + 3y \geq 5$. Use alphabetical order to replace x with 4 and y with -1. $x + 3y \geq 5$ $4 + 3(-1) \; ? \; 5$ $4 - 3$ 1 FALSE Since $1 \geq 5$ is false, $(4, -1)$ is not a solution.	10. Determine whether $(-2, 5)$ is a solution of $3x + y \leq -1$. A. Yes B. No

Objective [13.4b] Graph linear inequalities.

Brief Procedure

1. Replace the inequality symbol with an equals sign and graph this related equation.
2. If the inequality symbol is $<$ or $>$, draw the line dashed. If the inequality symbol is $\leq$ or $\geq$, draw the line solid.
3. The graph consists of a half-plane, either above or below or left or right of the line, and, if the line is solid, the line as well. To determine which half-plane to shade, choose a point not on the line as a test point. Substitute to find whether that point is a solution of the inequality. If it is, shade the half-plane containing that point. If it is not, shade the half-plane on the opposite side of the line.

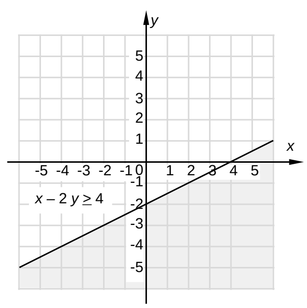
Example

Graph: $x - 2y \geq 4$.

First graph the line $x - 2y = 4$. The intercepts are $(0, -2)$ and (4,0). We draw the line solid since the inequality symbol is $\geq$. Next, choose a test point not on the line and determine if it is a solution of the inequality. We choose (0,0), since it is usually an easy point to use.

$$\begin{array}{c|c} \multicolumn{2}{c}{x - 2y \geq 4} \\ \hline 0 - 2 \cdot 0 & ?\ 4 \\ 0 & \quad \text{FALSE} \end{array}$$

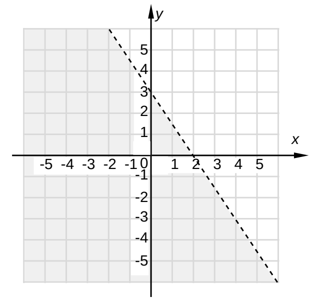
Since (0,0) is not a solution, we shade the half-plane that does not contain (0,0).



Practice Exercise

11. Graph $3x + 2y < -6$.

A.



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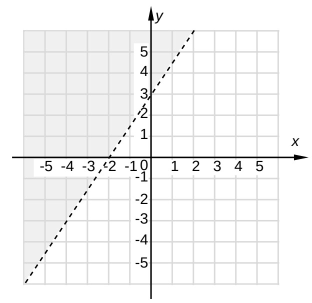

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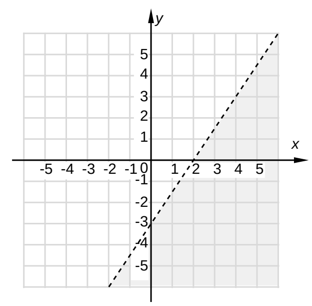

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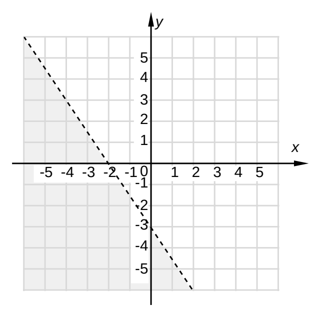

Objective [13.5a] Find an equation of direct variation given a pair of values of the variables.		
Brief Procedure	Example	Practice Exercise
An equation of direct variation has the form $y = kx$, where k is a positive constant. Substitute the given values in this equation to find k.	Find an equation of variation in which y varies directly as x and $y = 20$ when $x = 4$. We substitute to find k: $y = kx$ $20 = k \cdot 4$ $5 = k$ Then the equation of variation is $y = 5x$.	12. Find an equation of variation in which y varies directly as x and $y = 3$ when $x = 2$. A. $y = \frac{2}{3}x$ B. $y = \frac{3}{2}x$ C. $y = 5x$ D. $y = 6x$

Objective [13.5b] Solve applied problems involving direct variation.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process, translating to an equation of direct variation.	The interest I earned in 1 yr on a fixed principal varies directly as the interest rate r. An investment earns \$56.25 at an interest rate of 3.75%. How much will the investment earn at a rate of 4.5%? 1., 2. *Familiarize and Translate*. The problem states that we have direct variation between the variables I and r. Thus, an equation $I = kr$, $k > 0$, applies. As the interest rate increases, the amount of interest earned increases. 3. *Solve*. First find an equation of variation. $I = kr$ $56.25 = k \cdot 0.0375$ $\frac{56.25}{0.0375} = k$ $1500 = k$ The equation of variation is $I = 1500r$. Now use the equation to find the interest earned when the interest rate is 4.5%. $I = 1500r$ $I = 1500(0.045)$ $I = 67.50$ (continued)	13. The amount of Melissa's paycheck P varies directly as the number H of hours worked. For working 16 hr, her pay is \$132. Find her pay for 28 hr of work. A. \$224 B. \$231 C. \$242 D. \$256

Objective [13.5b] (continued)

Brief Procedure	Example	Practice Exercise
	4. *Check*. This check might be done by repeating the computations. We might also do some reasoning about the answer. The interest rate increased from 3.75% to 4.5%. Similarly, the interest earned increased from \$56.25 to \$67.50. 5. *State*. When the interest rate is 4.5%, the investment earns \$67.50.	

Objective [13.5c] Find an equation of inverse variation given a pair of values of the variables.

Brief Procedure	Example	Practice Exercise
An equation of inverse variation is of the form $y = k/x$, where k is a positive constant. Substitute the given values in the equation to find k.	Find an equation of variation in which y varies inversely as x and $y = 10$ when $x = 0.5$. We substitute to find k. $y = \frac{k}{x}$ $10 = \frac{k}{0.5}$ $5 = k$ The equation of variation is $y = \frac{5}{x}$.	14. Find an equation of variation in which y varies inversely as x and $y = 12$ when $x = 3$. A. $y = \frac{1}{36x}$ B. $y = \frac{1}{4x}$ C. $y = \frac{4}{x}$ D. $y = \frac{36}{x}$

Objective [13.5d] Solve applied problems involving inverse variation.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process, translating to an equation of inverse variation.	The time t required to drive a fixed distance varies inversely as the speed r. It takes 4 hr at 60 mph to drive a fixed distance. How long would it take at 50 mph? 1. *Familiarize*. The problem states that we have inverse variation between the variables t and r. As the speed decreases, the time required to travel the fixed distance increases. 2. *Translate*. We write an equation of variation. Travel time varies inversely as speed. This translates to $t = \frac{k}{r}$. (continued)	15. It takes 4 days for 2 people to paint a house. How long will it take 3 people to do the job? A. 2 days B. $2\frac{2}{3}$ days C. 3 days D. $3\frac{1}{3}$ days

Objective [13.5d] (continued)		
Brief Procedure	Example	Practice Exercise
	3. *Solve.* First find an equation of variation. $t = \frac{k}{r}$ $4 = \frac{k}{60}$ $240 = k$ The equation is $t = \frac{240}{r}$. Now use the equation to find the time required to travel the fixed distance at 50 mph. $t = \frac{240}{r}$ $t = \frac{240}{50}$ $t = 4.8$ 4. *Check.* In addition to repeating the computations, we can analyze the results. The speed decreased from 60 mph to 50 mph, and the travel time increased from 4 hr to 4.8 hr. This is what we would expect with inverse variation. 5. *State.* It would take 4.8 hr to travel the fixed distance at a speed of 50 mph.	