Developmental Mathematics
Chapter 12 Review

Objective [12.1a] Find all numbers for which a rational expression is undefined.		
Brief Procedure	Example	Practice Exercise
Determine the values of the variable that make the denominator zero.	Find all numbers for which the rational expression $\dfrac{x-2}{x^2-16}$ is undefined. We set the denominator equal to 0 and solve. $x^2-16=0$ $(x+4)(x-4)=0$ $x+4=0 \quad \textit{or} \quad x-4=0$ $x=-4 \quad \textit{or} \quad x=4$ The expression is undefined for the numbers -4 and 4.	1. Find all numbers for which the rational expression $\dfrac{y+3}{y^2+4y-5}$ is undefined. A. -5 B. -3 C. $-5, 1$ D. $-5, -3, 1$
Objective [12.1b] Multiply a rational expression by 1, using an expression such as A/A.		
Brief Procedure	Example	Practice Exercise
Multiply the numerators and multiply the denominators.	Multiply: $\dfrac{x+3}{2x-1} \cdot \dfrac{x+2}{x+2}$. $\dfrac{x+3}{2x-1} \cdot \dfrac{x+2}{x+2} = \dfrac{(x+3)(x+2)}{(2x-1)(x+2)}$	2. Multiply: $\dfrac{3y}{y-4} \cdot \dfrac{2y}{2y}$. A. $\dfrac{(3y)(2y)}{y-4}$ B. $\dfrac{3y}{(y-4)(2y)}$ C. $\dfrac{2y}{(y-4)(2y)}$ D. $\dfrac{(3y)(2y)}{(y-4)(2y)}$

Objective [12.1c] Simplify rational expressions by factoring the numerator and the denominator and removing factors of 1.

Brief Procedure	Example	Practice Exercise
Factor the numerator and the denominator of the rational expression and remove factors that are common to the numerator and the denominator. These factors of 1 can also be canceled.	Simplify by removing a factor of 1: $\frac{6x^2+9x}{3x^2-3x}$. $\frac{6x^2+9x}{3x^2-3x} = \frac{3x(2x+3)}{3x(x-1)}$ $= \frac{3x}{3x}\cdot\frac{2x+3}{x-1}$ $= 1\cdot\frac{2x+3}{x-1}$ $= \frac{2x+3}{x-1}$ This could be done using canceling as follows: $\frac{6x^2+9x}{3x^2-3x} = \frac{3x(2x+3)}{3x(x-1)}$ $= \frac{\cancel{3x}(2x+3)}{\cancel{3x}(x-1)}$ $= \frac{2x+3}{x-1}$	3. Simplify by removing a factor of 1: $\frac{x^2+x-6}{x^2+6x+9}$. A. $\frac{x-6}{6x+9}$ B. $\frac{x-2}{x+3}$ C. $-\frac{1}{3}$ D. $-\frac{2}{3}$

Objective [12.1d] Multiply rational expressions and simplify.

Brief Procedure	Example	Practice Exercise
Multiply the numerators and the denominators and then simplify by removing a factor of 1.	Multiply and simplify: $\frac{x^2+3x-4}{18}\cdot\frac{6}{x^2+x-12}$. $\frac{x^2+3x-4}{18}\cdot\frac{6}{x^2+x-12}$ $= \frac{(x^2+3x-4)6}{18(x^2+x-12)}$ $= \frac{(x+4)(x-1)6}{3(6)(x+4)(x-3)}$ $= \frac{\cancel{(x+4)}(x-1)\cancel{6}}{3(\cancel{6})\cancel{(x+4)}(x-3)}$ $= \frac{x-1}{3(x-3)}$	4. Multiply and simplify: $\frac{a+1}{a-3}\cdot\frac{a^2+2a-15}{a^2-a-2}$. A. $\frac{a+5}{a-2}$ B. $\frac{a-5}{a-2}$ C. $\frac{a+5}{a+2}$ D. $\frac{a-5}{a+2}$

Objective [12.2a] Find the reciprocal of a rational expression.		
Brief Procedure	Example	Practice Exercise
Interchange the numerator and the denominator of the expression.	Find the reciprocal of each expression. a) $\dfrac{x+1}{x^2+5}$ b) $2y+3$ c) $\dfrac{1}{z-6}$ a) The reciprocal of $\dfrac{x+1}{x^2+5}$ is $\dfrac{x^2+5}{x+1}$. b) Think of $2y+3$ as $\dfrac{2y+3}{1}$. Then the reciprocal is $\dfrac{1}{2y+3}$. c) The reciprocal of $\dfrac{1}{z-6}$ is $\dfrac{z-6}{1}$, or $z-6$.	5. Find the reciprocal of $\dfrac{x-3}{2x+5}$. A. $\dfrac{5}{3}$ B. $2x+5$ C. $\dfrac{1}{x-3}$ D. $\dfrac{2x+5}{x-3}$

Objective [12.2b] Divide rational expressions and simplify.		
Brief Procedure	Example	Practice Exercise
Multiply by the reciprocal of the divisor. Then simplify by removing a factor of 1, if possible.	Divide and simplify: $\dfrac{y^2-4y+3}{y+6} \div \dfrac{2y-6}{y^2+2y-24}$. $\dfrac{y^2-4y+3}{y+6} \div \dfrac{2y-6}{y^2+2y-24}$ $= \dfrac{y^2-4y+3}{y+6} \cdot \dfrac{y^2+2y-24}{2y-6}$ $= \dfrac{(y^2-4y+3)(y^2+2y-24)}{(y+6)(2y-6)}$ $= \dfrac{(y-1)(y-3)(y+6)(y-4)}{(y+6)(2)(y-3)}$ $= \dfrac{(y-1)\cancel{(y-3)}\cancel{(y+6)}(y-4)}{\cancel{(y+6)}(2)\cancel{(y-3)}}$ $= \dfrac{(y-1)(y-4)}{2}$	6. Divide and simplify: $\dfrac{x^2-9}{5} \div \dfrac{x+3}{x-5}$. A. $x(x-3)$ B. $-\dfrac{3(x^2-9)}{25}$ C. $\dfrac{(x-3)(x-5)}{5}$ D. $\dfrac{(x+3)(x-5)}{5}$

Objective [12.3a] Find the LCM of several numbers by factoring.		
Brief Procedure	Example	Practice Exercise
Factor the numbers into prime factors and use each factor the greatest number of times that it occurs in any one factorization.	Find the LCM of 18 and 24. $18 = 2 \cdot 3 \cdot 3$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ The LCM is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$, or 72.	7. Find the LCM of 12 and 27. A. 36 B. 54 C. 108 D. 324

Objective [12.3b] Add fractions, first finding the LCD.		
Brief Procedure	Example	Practice Exercise
a) Find the least common multiple of the denominators. That number is the least common denominator, LCD. b) Multiply by 1, using an appropriate notation, n/n, to express each number in terms of the LCD. c) Add the numerators, keeping the same denominator. d) Simplify, if possible.	Add and simplify, if possible: $\frac{2}{9}+\frac{1}{6}$. $9=3\cdot 3$ and $6=2\cdot 3$ so the LCM of 9 and 6 is $2\cdot 3\cdot 3$, or 18. Thus the LCD is 18. $\frac{2}{9}+\frac{1}{6}$ $=\frac{2}{9}\cdot\frac{2}{2}+\frac{1}{6}\cdot\frac{3}{3}$ $=\frac{4}{18}+\frac{3}{18}$ $=\frac{7}{18}$ No simplification is necessary.	8. Add and simplify, if possible: $\frac{3}{4}+\frac{3}{10}$. A. $\frac{3}{7}$ B. $\frac{3}{14}$ C. $\frac{21}{20}$ D. $\frac{9}{40}$

Objective [12.3c] Find the LCM of algebraic expressions by factoring.		
Brief Procedure	Example	Practice Exercise
Factor each expression and use each factor the greatest number of times it occurs in any one factorization.	Find the LCM of $5x-5$ and x^2-2x+1. $5x-5=5(x-1)$ $x^2-2x+1=(x-1)(x-1)$ The LCM is $5(x-1)(x-1)$.	9. Find the LCM of y^2-9 and y^2+y-6. A. $(y+3)(y-3)(y-2)$ B. $(y+3)(y-3)(y+2)$ C. $(y+3)(y+3)(y-3)(y-2)$ D. $(y+3)(y-3)(y-3)(y-2)$

Objective [12.4a] Add rational expressions.		
Brief Procedure	Example	Practice Exercise
To add when the denominators are the same, add the numerators and keep the same denominator. To add when when the denominators are different: 1. Find the LCM of the denominators. This is the least common denominator (LCD). 2. For each rational expression find an equivalent expression with the LCD. To do so, multiply by 1 using an expression for 1 made up of factors of the LCD that are missing from the original denominator. 3. Add the numerators. Write the sum over the LCD. 4. Simplify, if possible.	Add: $\dfrac{2x}{3x+6}+\dfrac{1}{x^2-4}$. First we find the LCD: $3x+6=3(x+2)$ $x^2-4=(x+2)(x-2)$ The LCD is $3(x+2)(x-2)$. Then we have: $\dfrac{2x}{3(x+2)}\cdot\dfrac{x-2}{x-2}+$ $\dfrac{1}{(x+2)(x-2)}\cdot\dfrac{3}{3}$ $=\dfrac{2x(x-2)+3}{3(x+2)(x-2)}$ $=\dfrac{2x^2-4x+3}{3(x+2)(x-2)}$	10. Add: $\dfrac{3}{x^2-3x-4}+\dfrac{5}{x^2+3x+2}$ A. $\dfrac{8}{(x+1)(x-4)(x+2)}$ B. $\dfrac{3x+6}{(x+1)(x-4)(x+2)}$ C. $\dfrac{5x-20}{(x+1)(x-4)(x+2)}$ D. $\dfrac{8x-14}{(x+1)(x-4)(x+2)}$

Objective [12.5a] Subtract rational expressions.		
Brief Procedure	Example	Practice Exercise
To subtract when the denominators are the same, subtract the numerators and keep the same denominator. To subtract when denominators are different: 1. Find the LCM of the denominators. This is the least common denominator (LCD). 2. For each rational expression find an equivalent expression with the LCD. To do so, multiply by 1 using an expression for 1 made up of factors of the LCD that are missing from the original denominator. 3. Subtract the numerators. Write the sum over the LCD. 4. Simplify, if possible.	Subtract: $\dfrac{x-3}{x+5}-\dfrac{x-2}{x+1}$. The LCD is $(x+5)(x+1)$. $\dfrac{x-3}{x+5}-\dfrac{x-2}{x+1}$ $=\dfrac{x-3}{x+5}\cdot\dfrac{x+1}{x+1}-\dfrac{x-2}{x+1}\cdot\dfrac{x+5}{x+5}$ $=\dfrac{(x-3)(x+1)}{(x+5)(x+1)}-\dfrac{(x-2)(x+5)}{(x+1)(x+5)}$ $=\dfrac{x^2-2x-3}{(x+5)(x+1)}-\dfrac{x^2+3x-10}{(x+5)(x+1)}$ $=\dfrac{x^2-2x-3-(x^2+3x-10)}{(x+5)(x+1)}$ $=\dfrac{x^2-2x-3-x^2-3x+10}{(x+5)(x+1)}$ $=\dfrac{-5x+7}{(x+5)(x+1)}$	11. Subtract: $\dfrac{4}{x^2-36}-\dfrac{1}{x-6}$. A. $\dfrac{3}{(x+6)(x-6)}$ B. $\dfrac{-x+10}{(x+6)(x-6)}$ C. $\dfrac{-x-2}{(x+6)(x-6)}$ D. $\dfrac{x-2}{(x+6)(x-6)}$

Objective [12.5b] Simplify combined additions and subtractions of rational expressions.		
Brief Procedure	Example	Practice Exercise
Add and subtract as indicated and then simplify, if possible.	Perform the indicated operations and simplify, if possible: $\frac{4a}{a^2-4}-\frac{3}{a-2}+\frac{5}{a}.$ The LCD is $a(a+2)(a-2)$. $\frac{4a}{(a+2)(a-2)}\cdot\frac{a}{a}-$ $\frac{3}{a-2}\cdot\frac{a(a+2)}{a(a+2)}+$ $\frac{5}{a}\cdot\frac{(a+2)(a-2)}{(a+2)(a-2)}$ $=\frac{4a^2}{a(a+2)(a-2)}-$ $\frac{3a(a+2)}{a(a+2)(a-2)}+$ $\frac{5(a+2)(a-2)}{a(a+2)(a-2)}$ $=\frac{4a^2}{a(a+2)(a-2)}-$ $\frac{3a^2+6a}{a(a+2)(a-2)}+$ $\frac{5(a^2-4)}{a(a+2)(a-2)}$ $=\frac{4a^2-(3a^2+6a)+5a^2-20}{a(a+2)(a-2)}$ $=\frac{4a^2-3a^2-6a+5a^2-20}{a(a+2)(a-2)}$ $=\frac{6a^2-6a-20}{a(a+2)(a-2)}$	12. Perform the indicated operations and simplify, if possible: $\frac{3y}{y^2+y-20}+\frac{2}{y+5}-\frac{3}{y-4}.$ A. $\frac{-2y-7}{(y+5)(y-4)}$ B. $\frac{-2y-23}{(y+5)(y-4)}$ C. $\frac{2y-7}{(y+5)(y-4)}$ D. $\frac{2y-23}{(y+5)(y-4)}$

Objective [12.6a] Solve rational equations.

Brief Procedure

First multiply on both sides of the equation by the LCM of all the denominators to clear fractions. Then solve the resulting equation. Since this equation might have solutions that are not solutions of the original equation, the possible solutions *must* be checked in the *original* equation.

Example

Solve: $\frac{2x+1}{x-2} = \frac{x-1}{3x+2}$.

The LCM of the denominators is $(x-2)(3x+2)$. We multiply by the LCM on both sides.

$$\frac{2x+1}{x-2} = \frac{x-1}{3x+2}$$

$$(x-2)(3x+2) \cdot \frac{2x+1}{x-2} = (x-2)(3x+2) \cdot \frac{x-1}{3x+2}$$

$$(3x+2)(2x+1) = (x-2)(x-1)$$

$$6x^2 + 7x + 2 = x^2 - 3x + 2$$

$$5x^2 + 10x = 0$$

$$5x(x+2) = 0$$

$$5x = 0 \quad \textit{or} \quad x+2 = 0$$

$$x = 0 \quad \textit{or} \quad x = -2$$

Both numbers check. The solutions are 0 and -2.

Practice Exercise

13. Solve: $x - \frac{8}{x} = 2$.

A. -2

B. 4

C. $-2, 4$

D. $-2, 4, 8$

Objective [12.7a] Solve applied problems using rational equations.

Brief Procedure

Use the five-step problem solving process.

Example

One number is 3 more than another. The quotient of the larger number divided by the smaller is $\frac{3}{2}$. Find the numbers.

1. *Familiarize.* Let x = the smaller number. Then $x + 3$ = the larger number and the quotient of the larger divided by the smaller is $\frac{x+3}{x}$.
2. *Translate.*

$$\begin{array}{ccc} \underbrace{\text{The quotient}} & \text{is} & \frac{3}{2}. \\ \downarrow & \downarrow & \downarrow \\ \frac{x+3}{x} & = & \frac{3}{2} \end{array}$$

3. *Solve.* We solve the equation.

$$\frac{x+3}{x} = \frac{3}{2}$$

$$2x \cdot \frac{x+3}{x} = 2x \cdot \frac{3}{2}$$

$$2(x+3) = 3x$$

$$2x + 6 = 3x$$

$$6 = x$$

If $x = 6$, then $x + 3 = 6 + 3$, or 9.

(continued)

Practice Exercise

14. A passenger car travels 10 km/h faster than a delivery van. While the car travels 240 km, the van travels 200 km. Find their speeds.

A. The speed of the car is 45 km/h.

B. The speed of the car is 50 km/h.

C. The speed of the car is 60 km/h.

D. The speed of the car is 65 km/h.

Objective [12.7a] (continued)		
Brief Procedure	Example	Practice Exercise
	4. *Check.* The larger number, 9, is 3 more than the smaller number, 6. Also $\frac{9}{6} = \frac{3}{2}$, so the numbers check. 5. *State.* The numbers are 6 and 9.	

Objective [12.7b] Solve proportion problems.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process, translating to a proportion (an equality of ratios).	Melinda biked 78 mi in 5 days. At this rate, how far would she bike in 7 days? 1. *Familiarize.* We can set up ratios, letting m = the number of miles Melinda would bike in 7 days. 2. *Translate.* We assume Melinda bikes at the same rate during the entire 7 days. Thus, the ratios are the same and we can write a proportion. $\begin{array}{r} \text{Miles} \rightarrow \\ \text{Days} \rightarrow \end{array} \frac{78}{5} = \frac{m}{7} \begin{array}{l} \leftarrow \text{Miles} \\ \leftarrow \text{Days} \end{array}$ 3. *Solve.* We multiply by the LCM, $5 \cdot 7$, or 35, on both sides. $35 \cdot \frac{78}{5} = 35 \cdot \frac{m}{7}$ $7 \cdot 78 = 5 \cdot m$ $\frac{7 \cdot 78}{5} = m$ $109.2 = m$ 4. *Check.* $\frac{78}{5} = 15.6$ and $\frac{109.2}{7} = 15.6$; since the ratios are the same, the answer checks. 5. *State.* Melinda would bike 109.2 mi in 7 days.	15. Jeremy can read 6 pages of his history textbook in 20 min. At this rate, how many pages can he read in 50 min? A. 13 B. 15 C. 18 D. 20

Objective [12.8a] Solve a rational formula for a letter.		
Brief Procedure	**Example**	**Practice Exercise**
Identify the letter and: 1. Multiply on both sides to clear fractions or decimals, if necessary. 2. Multiply to remove parentheses, if necessary. 3. Get all terms with the letter to be solved for on one side of the equation and all other terms on the other side using the addition principle. 4. Factor out the unknown, if necessary. 5. Solve for the letter in question, using the multiplication principle.	Solve $S = \frac{n}{2}(a+l)$ for l. $S = \frac{n}{2}(a+l)$ $2S = n(a+l)$ $2S = an + ln$ $2S - an = ln$ $\frac{2S-an}{n} = l$, or $\frac{2S}{n} - a = l$	16. Solve $f = \frac{kMm}{d^2}$ for M. A. $M = fd^2km$ B. $M = \frac{fkm}{d^2}$ C. $M = \frac{fd^2}{km}$ D. $M = \frac{f}{d^2km}$

Objective [12.9a] Simplify complex rational expressions.		
Brief Procedure	**Example**	**Practice Exercise**
To simplify by multiplying by the LCM of all the denominators: 1. First, find the LCM of all the denominators of all the rational expressions occurring *within* both the numerator and the denominator of the complex rational expression. 2. Then multiply by 1 using LCM/LCM. 3. If possible, simplify by removing a factor of 1. To simplify by adding or subtracting in the numerator and in the denominator: 1. Add or subtract, as necessary, to get a single rational expression in the numerator. 2. Add or subtract, as necessary, to get a single rational expression in the denominator. 3. Divide the numerator by the denominator. 4. If possible, simplify by removing a factor of 1.	Simplify: $\dfrac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$. Using the first method described at the left, we first observe that the LCM of the denominators within the numerator and the denominator is x^2. We multiply by 1 using x^2/x^2. $\dfrac{1-\frac{1}{x}}{1-\frac{1}{x^2}} \cdot \dfrac{x^2}{x^2}$ $= \dfrac{\left(1-\frac{1}{x}\right)x^2}{\left(1-\frac{1}{x^2}\right)x^2}$ $= \dfrac{1\cdot x^2 - \frac{1}{x}\cdot x^2}{1\cdot x^2 - \frac{1}{x^2}\cdot x^2}$ $= \dfrac{x^2-x}{x^2-1}$ $= \dfrac{x(x-1)}{(x+1)(x-1)}$ $= \dfrac{x\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$ $= \dfrac{x}{x+1}$ (continued)	17. Simplify: $\dfrac{\frac{3}{y}+\frac{1}{y}}{y-\frac{y}{3}}$. A. $\frac{20}{9}$ B. $\frac{10}{9}$ C. $\frac{6}{y}$ D. $\frac{6}{y^2}$

Objective [12.9a] (continued)		
Brief Procedure	Example	Practice Exercise
	Using the second method, we first subtract in the numerator and in the denominator. $$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}} = \frac{1\cdot\frac{x}{x}-\frac{1}{x}}{1\cdot\frac{x^2}{x^2}-\frac{1}{x^2}} = \frac{\frac{x}{x}-\frac{1}{x}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \frac{\frac{x-1}{x}}{\frac{x^2-1}{x^2}} = \frac{x-1}{x}\cdot\frac{x^2}{x^2-1} = \frac{(x-1)(x)(x)}{x(x+1)(x-1)} = \frac{\cancel{(x-1)}(\cancel{x})(x)}{\cancel{x}(x+1)\cancel{(x-1)}} = \frac{x}{x+1}$$	