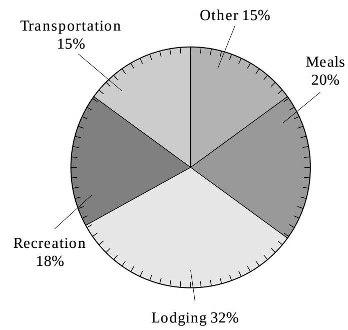
Developmental Mathematics
Chapter 9 Review

Objective [9.1a] Solve applied problems involving circle, bar, and line graphs.

Brief Procedure	Example
To solve an applied problem involving a circle graph, examine the graph carefully, noting the items listed and the percents.	The following circle graph shows how vacation money is spent.



Suppose a family spends \$2000 on a vacation. How much is spent for transportation?

1. *Familiarize.* The graph shows that 15% of vacation money is spent on transportation. Let $t =$ the amount spent on transportation.
2. *Translate.* We reword the problem and translate.

   What is 15% of \$2000?

   $$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ t & = & 15\% & \cdot & 2000 \end{array}$$

3. *Solve.* We carry out the computation.

   $t = 15\% \cdot 2000 = 0.15 \cdot 2000 = 300$

4. *Check.* We repeat the calculation. The answer checks.
5. *State.* \$300 is spent for transportation.

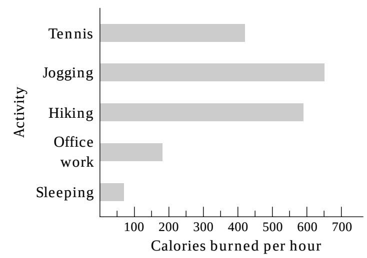
Practice Exercise

1. Suppose a family spends \$800 on a vacation. Use the circle graph in the example above to determine how much is spent for meals.

A. \$120
B. \$144
C. \$160
D. \$256

Objective [9.1a] continued

Brief Procedure	Example
To solve an applied problem involving a bar graph, examine the graph carefully, noting the items listed, the scale used, and the lengths of the bars.	The following bar graph shows the number of calories burned per hour by a 152 lb person during various activities.



Which activity burns the fewest calories per hour?

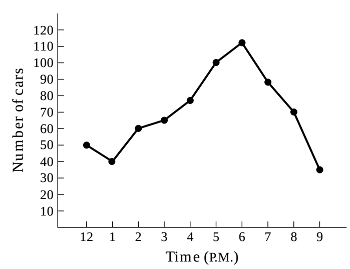
The shortest bar is for sleeping. Thus, sleeping burns the fewest calories.

Practice Exercise

2. Use the bar graph in the example above to determine which activity burns about 420 calories per hour.

A.Tennis
B. Jogging
C. Hiking
D. Office work

	Example
To solve an applied problem involving a line graph, examine the graph carefully, noting the items on the horizontal and vertical scales, the marks on the scales, and the points on the graph.	The following line graph shows the number of cars passing through an intersection during various hours of the day.



During which hour was traffic the heaviest?

Find the highest point on the graph and then go down to the horizontal scale to read the corresponding hour. We see that traffic was heaviest during the 6 P.M. hour.

Practice Exercise

3. During which hour did about 70 cars pass through the intersection?

A. The 2 P.M. hour
B. The 4 P.M. hour
C. The 6 P.M. hour
D. The 8 P.M. hour

Objective [9.1b] Plot points associated with ordered pairs of numbers.

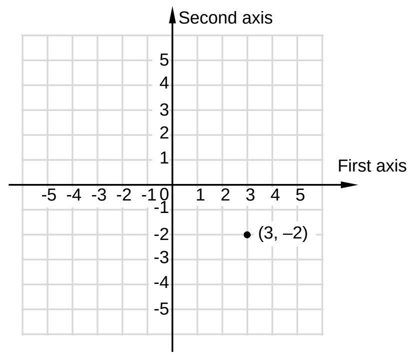
Brief Procedure

Given a point (a, b), start at the origin and move a units right or left depending on whether a is positive or negative. Then move b units up or down depending on whether b is positive or negative. Make a dot and label the point.

Example

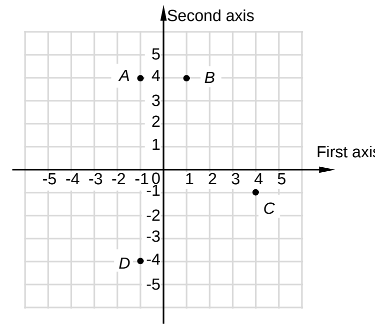
Plot the point $(3, -2)$.

The first coordinate is positive so, starting at the origin, move 3 units to the right. The second coordinate is negative, so we then move down 2 units.



Practice Exercise

4. Which point is $(-1, 4)$?

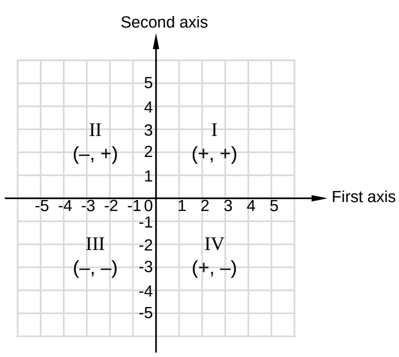


A. A
B. B
C. C
D. D

Objective [9.1c] Determine the quadrant in which a point lies.

Brief Procedure

The following figure shows the signs of coordinates of points in each quadrant.



Example

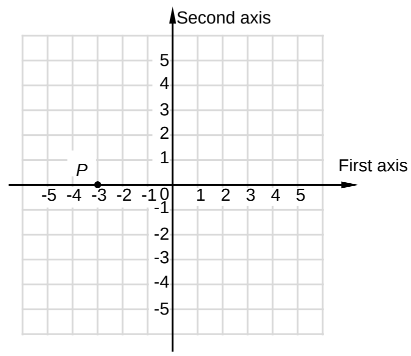
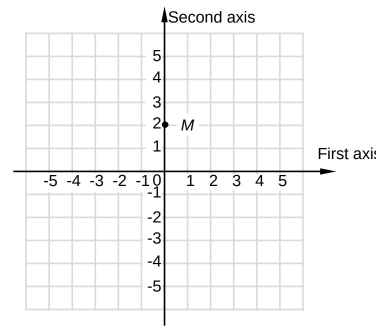
In which quadrant is the point $(-3, -5)$ located?

Both coordinates are negative, so $(-3, -5)$ is in quadrant III.

Practice Exercise

5. In which quadrant is the point $(2, -1)$ located?
A. I
B. II
C. III
D. IV

Objective [9.1d] Find the coordinates of a point on a graph.

Brief Procedure	Example	Practice Exercise
Determine how far the point is to the right or left of the origin and then how far up or down.	Find the coordinates of point P.  Point P is 3 units to the left of the origin and 0 units up or down. Its coordinates are $(-3, 0)$.	6. Find the coordinates of point M.  A. $(0, -2)$ B. (0,2) C. (2,0) D. (2,2)

Objective [9.2a] Determine whether an ordered pair is a solution of an equation with two variables.

Brief Procedure	Example	Practice Exercise
Substitute coordinates of the ordered pair for the variables, using the first number to replace the variable that occurs first alphabetically. If a true equation results, the pair is a solution.	Determine whether $(-2, 2)$ is a solution of $2b - a = 6$. We substitute -2 for a and 2 for b. $2b - a = 6$ $2 \cdot 2 - (-2) \ ? \ 6$ $4 + 2$ 6 TRUE Since $6 = 6$ is true, $(-2, 2)$ is a solution of the equation.	7. Determine whether $(-4, 1)$ is a solution of $n - m = -5$. A. Yes B. No

Objective [9.2b] Graph linear equations of the type $y = mx + b$ and $Ax + By = C$, identifying the y-intercept.

Brief Procedure

1. Select a value for one variable and calculate the corresponding value of the other variable. Form an ordered pair using alphabetical order as indicated by the variables.
2. Repeat step (1) to obtain at least two other ordered pairs. Two points are essential to determine a straight line. A third point serves a a check.
3. Plot the ordered pairs and draw a straight line passing through the points.

If the equation is given in the the form $Ax + By = C$, it is often convenient to solve for y before using the procedure above.

Given an equation in the form $y = mx + b$, the y-intercept is $(0, b)$.

Example

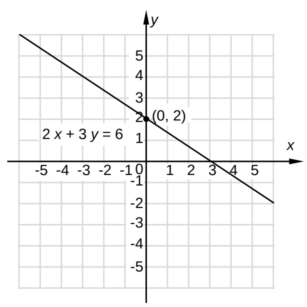
Graph $2x + 3y = 6$ and identify the y-intercept.

First we solve for y to find an equivalent equation in the form $y = mx + b$.

$$2x + 3y = 6$$
$$3y = -2x + 6$$
$$\frac{1}{3} \cdot 3y = \frac{1}{3}\Big(-2x + 6\Big)$$
$$y = -\frac{2}{3}x + 2$$

From the last equation we see that the y-intercept is (0,2). We find two other pairs that are solutions, using multiples of 3 to avoid fractions. We then complete and label the graph.

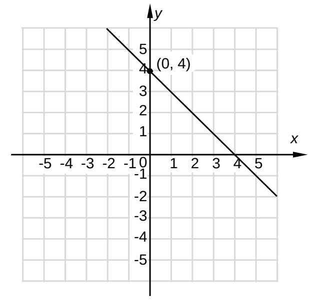
x	y
0	2
-3	4
3	0



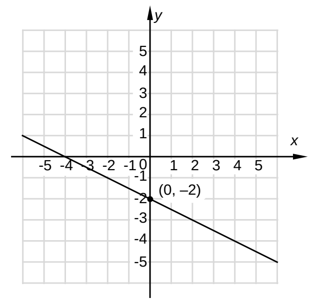
Practice Exercise

8. Graph $x - 2y = 4$ and label the y-intercept.

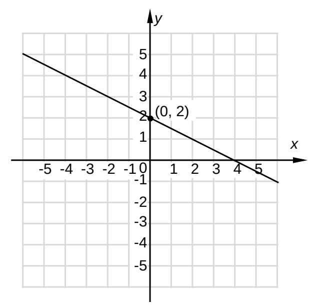
A.



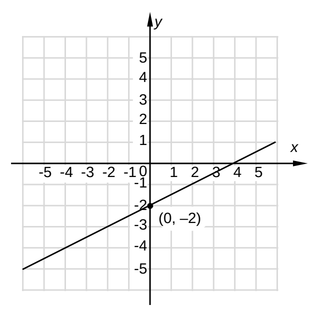
B.

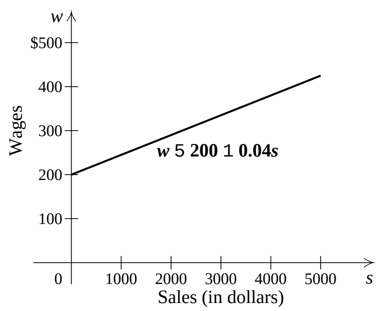


C.



D.



Objective [9.2c] Solve applied problems involving graphs of linear equations.		
Brief Procedure	Example	Practice Exercise
Given a linear equation that represents a real-world situation, we can substitute in the equation for one variable to find values of the other variable. Then we can graph the equation and use the graph to obtain information.	The weekly salary of a salesperson at Shoe City is given by the equation $w = 200 + 0.04s$, where s = that person's sales for the week. Graph the equation and then use the graph to estimate a salesperson's sales when the week's pay is \$375. We choose some values for s and find the corresponding w-values. When $s = 1000$, $w = 200 + 0.04(1000) = 240$. When $s = 3000$, $w = 200 + 0.04(3000) = 320$. When $s = 5000$, $w = 200 + 0.04(5000) = 400$. Plot these points and draw the graph.  To estimate the sales when the week's pay is \$375, locate 375 on the w-axis, go across horizontally to the graph, and then go down vertically to the s-axis. We find that sales are about \$4400 when a week's pay is \$375.	9. The cost c, in dollars, of renting a 20-ft moving van at Rent King is given by the equation $c = 0.45m + 59.95$, where m = the number of miles the truck is driven. Graph the equation and then use the graph to estimate how far a van can be driven on a budget of \$150. A. About 60 miles B. About 150 miles C. About 200 miles D. About 270 miles

Objective [9.3a] Find the intercepts of a linear equation and graph using intercepts.

Brief Procedure

The x-intercept has the form $(a, 0)$. To find a, let $y = 0$ and solve the original equation for x.
The y-intercept has the form $(0, b)$. To find b, let $x = 0$ and solve the original equation for y.

To graph using intercepts, plot the intercepts and draw the line containing them. As a check that the graph is correct, find a third solution of the equation. If it is on the graph, then the graph is probably correct.

Example

Find the intercepts of $5x - 3y = 15$. Then use the intercepts to graph the equation.

To find the x-intercept, let $y = 0$. Then solve for x.

$$5x - 3y = 15$$
$$5x - 3 \cdot 0 = 15$$
$$5x = 15$$
$$x = 3$$

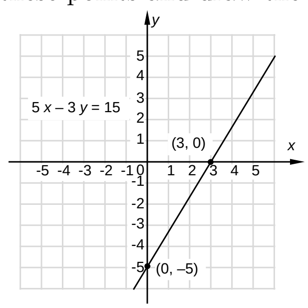
Thus, $(3, 0)$ is the x-intercept.

To find the y-intercept, let $x = 0$. Then solve for y.

$$5x - 3y = 15$$
$$5 \cdot 0 - 3y = 15$$
$$-3y = 15$$
$$y = -5$$

Thus, $(0, -5)$ is the y-intercept.

Plot these points and draw the line.



A third point should be used as a check. We substitute any value for x and solve for y.

We let $x = 6$. Then

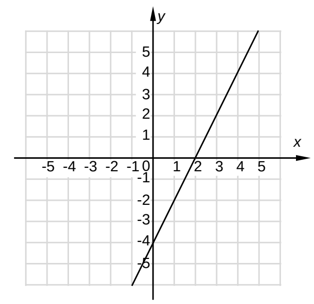
$$5x - 3y = 15$$
$$5 \cdot 6 - 3y = 15$$
$$30 - 3y = 15$$
$$-3y = -15$$
$$y = 5$$

The point $(6, 5)$ is on the graph, so the graph is probably correct.

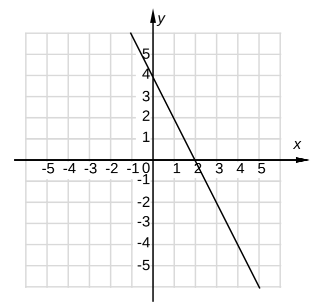
Practice Exercise

10. Find the intercepts of $2x - y = 4$. Then use the intercepts to graph the equation.

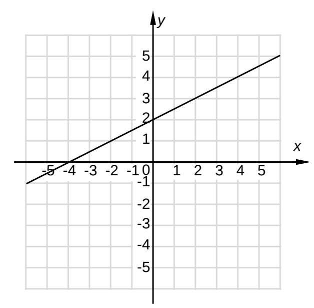
A.



B.



C.



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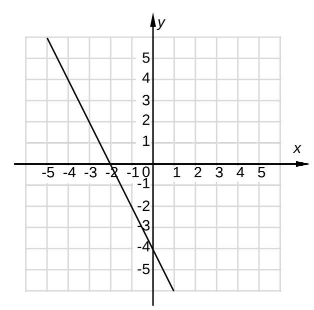


Objective [9.3b] Graph equations equivalent to those of the type $x = a$ and $y = b$.

Brief Procedure

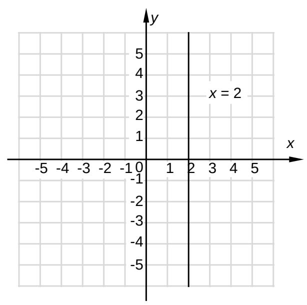
The graph of $x = a$ is a vertical line.

Example

Graph $x = 2$.

We can think of this equation as $x + 0 \cdot y = 2$. No matter what number we choose for y, x must be 2. We make a table of values and plot and connect the corresponding points.

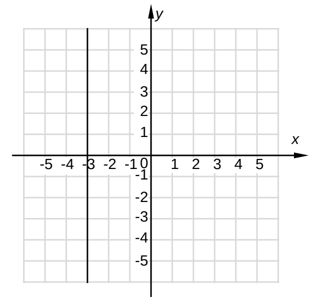
x	y
2	-4
2	0
2	3



Practice Exercise

11. Graph $x = -3$.

A.



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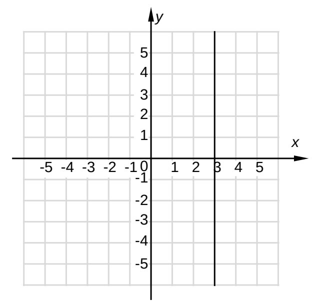
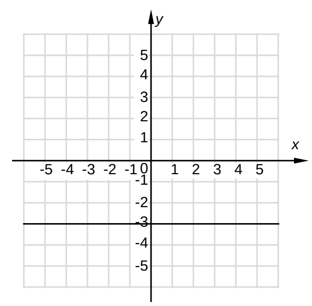
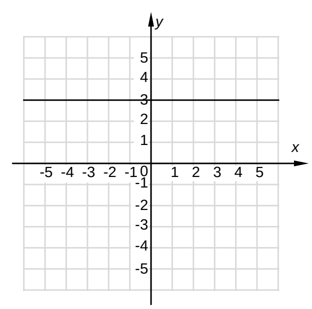


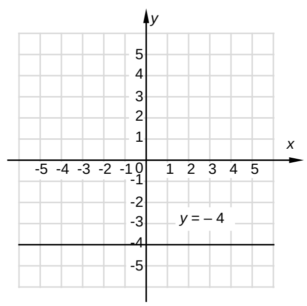
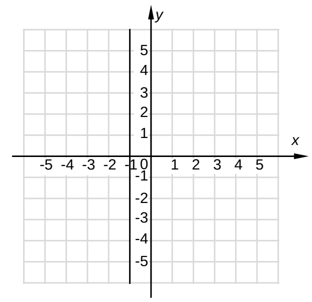
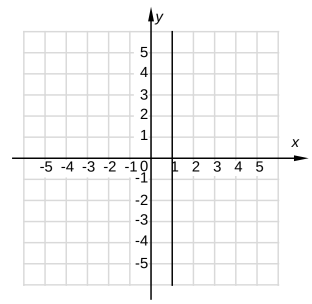
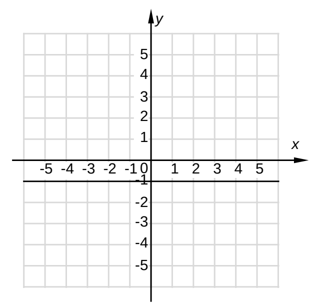
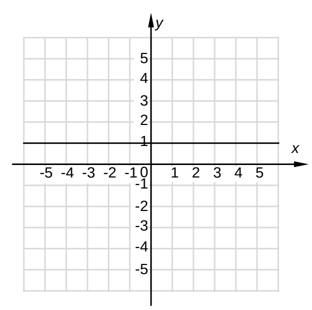
C.



D.



Objective [9.3b] (continued)

Brief Procedure	Example	Practice Exercise
The graph of $y = b$ is a horizontal line.	Graph $y = -4$. We can think of this equation as $0 \cdot x + y = -4$. No matter what number we choose for x, y must be -4. We make a table of values and plot and connect the corresponding points. x | y -2 | -4 0 | -4 3 | -4  	12. Graph $y = 1$. A.  B.  C.  D. 

Objective [9.4a] Find the mean (average), the median, and the mode of a set of data and solve related applied problems.

Brief Procedure	Example	Practice Exercise
To find the mean, or average, of a set of numbers, add the numbers and then divide by the number of addends.	A student's scores on four tests were 80, 64, 91, and 85. What was the average score? $\frac{80+64+91+85}{4} = \frac{320}{4} = 80$ The average score was 80.	13. On 5 successive days, Morgan ran 4 mi, 2 mi, 10 mi, 3 mi, and 6 mi. What was the average number of miles per day? A. 4.5 mi B. 5 mi C. 6.25 mi D. 7 mi
To find the median of a set of data, list the data in order from smallest to largest. The median is the middle number if there is an odd number of data items. If there is an even number of data items, the median is the average of the two middle numbers.	Find the median of each set of hourly wages. a) $6.50, $5.75, $7.25, $8.00, $7.40 b) $20, $15, $10, $12 a) List the data in order from smallest to largest: $5.75, $6.50, $7.25, $7.40, $8.00 There is an odd number of data items. The middle number is $7.25, so the median wage is $7.25. b) List the data in order from smallest to largest. $10, $12, $15, $20 There is an even number of items. The median is the average of the two middle numbers: $\text{Median} = \frac{\$12+\$15}{2} = \frac{\$27}{2} = \$13.50$	14. Find the median of the following temperatures: 56°, 48°, 61°, 66°, 53° A. 53° B. 56° C. 58.5° D. 61°
The mode of a set of data is the number or numbers that occur most often. If each number occurs the same number of times, there is no mode.	Find the modes of each set of data. a) 16, 23, 27, 27, 27 b) $34, $34, $51, $58, $58, $64 c) 7, 9, 15, 21, 45 a) The number that occurs most often is 27. Thus the mode is 27. b) The two numbers $34 and $58 occur most often. Thus the modes are $34 and $58. c) No number occurs more often than any other. Thus there is no mode.	15. Find the mode of these data: $17, $28, $33, $41, $56, $56, $91 A. $41 B. $46 C. $56 D. There is no mode.

Objective [9.4b] Compare two sets of data using their means.

Brief Procedure	Example
Find the mean, or average, of each set of data and compare the results.	Volunteers drank two brands of orange juice and rated their taste from 1 to 10, where 10 represents the best taste. The results are given below. On the basis of this test, which brand tastes better? Brand A: 7, 8, 6, 4, 10, 5, 9, 8, 8, 7 Brand B: 6, 10, 9, 7, 8, 7, 4, 5, 6, 7 Brand A average: $\frac{7+8+6+4+10+5+9+8+8+7}{10} = \frac{72}{10} = 7.2$ Brand B average: $\frac{6+10+9+7+8+7+4+5+6+7}{10} = \frac{69}{10} = 6.9$ The average for Brand A is higher than that for Brand B, so Brand A tastes better.
	Practice Exercise
	16. Two brands of light bulbs were tested. The lives, in hours, of 8 bulbs of each brand are listed below. On the basis of this test, which bulb is better? Brand A: 950, 967, 835, 1214, 1130, 891, 1070, 998 Brand B: 1015, 898, 1147, 935, 946, 893, 1235, 842 A. Brand A B. Brand B

Objective [9.4c] Make predictions from a set of data using interpolation or extrapolation.

Brief Procedure

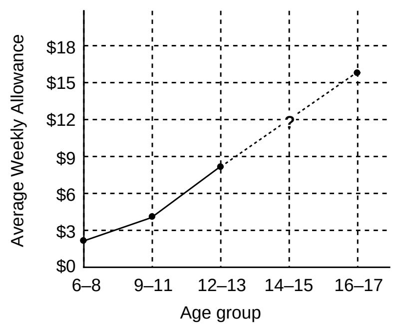
Interpolation can be used to find a value between two known values. To use interpolation to make a prediction, we can graph the given data and read the predicted value from the graph. We can also find the average of the known values on either side of the missing value.

Example

The following table gives the average weekly allowance for children in various age groups. Use interpolation to estimate the average weekly allowance for children in the 14-15 age group.

Age Group	Allowance
6-8	\$2.79
9-11	\$4.08
12-13	\$8.16
14-15	?
16-17	\$15.70

First we graph the data.



From the graph we estimate that the average weekly allowance for children in the 14-15 age group is about \$12.

We can also estimate this value by finding the average of the data values \$8.16 and \$15.70:

$$\frac{\$8.16 + \$15.70}{2} = \$11.93$$

Practice Exercise

17. The following table gives the times several math students spent studying for a test and their test scores. Estimate the missing data value.

Study time (in hours)	Test score (in percent)
4	76
6	79
7	80
9	85
10	?
11	91

A. 86
B. 88
C. 90
D. 92

Objective [9.4c] continued

Brief Procedure

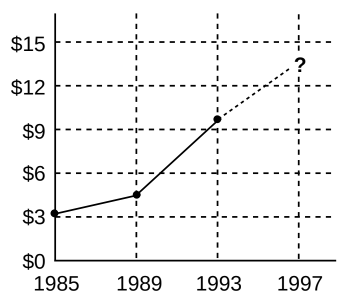
Extrapolation can be used to find a value that goes beyond the given data. To use extrapolation to make a prediction, we graph the data, extend the graph, and read the predicted value from the extended graph.

Example

The following table gives the average weekly allowance of children 12 years old and younger in various years. Use extrapolation to estimate the income in 1997.

Year	Income
1985	\$3.03
1989	\$4.42
1993	\$9.56
1997	?

We graph the given data and then draw a "representative" line beyond the data.



From the graph we estimate that the value for 1997 is about \$13.50. Answers will vary depending on the placement of the "representative" line.

Practice Exercise

18. The following table gives the prices of 2″ x 4″ lumber of various lengths. Use extrapolation to estimate the price of an 18-ft piece of 2″ x 4″ lumber.

Length	Price
8 ft	\$1.99
10 ft	\$2.99
12 ft	\$3.78
14 ft	\$4.57
16 ft	\$5.98
18 ft	?

A. About \$7
B. About \$9
C. About \$10
D. About \$12