Developmental Mathematics
Chapter 4 Review

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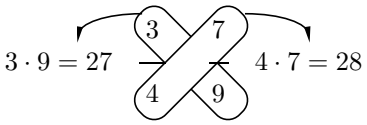
<tr><td colspan="3">Objective [4.1a] Find fractional notation for ratios.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Write the ratio of a to b as $\frac{a}{b}$.</td><td>Find the ratio of 5 to 12.
Write a fraction with a numerator of 5 and a denominator of 12: $\frac{5}{12}$.</td><td>1. Find the ratio of 8 to 3.
A. $\frac{3}{8}$
B. $\frac{8}{3}$
C. 8, 3
D. 83</td></tr>
<tr><td colspan="3">Objective [4.1b] Give the ratio of two different kinds of measure as a rate.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Divide the first measure by the second.</td><td>A driver travels 132 mi on 5.5 gal of gas. What is the rate in miles per gallon?
$\frac{132 \text{ mi}}{5.5 \text{ gal}} = 24 \frac{\text{mi}}{\text{gal}}$</td><td>2. A student earned \$91 for working 14 hr. What was the rate of pay per hour?
A. \$5.25 per hour
B. \$5.75 per hour
C. \$6.50 per hour
D. \$7.50 per hour</td></tr>
<tr><td colspan="3">Objective [4.1c] Determine whether two pairs of numbers are proportional.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Write each pair of numbers as a ratio, using fractional notation. Then use cross products to determine if the ratios are the same. If they are, the numbers are proportional.</td><td>Determine whether 3, 4 and 7, 9 are proportional.
 

Since the cross products are not the same ($27 \neq 28$), then $\frac{3}{4} \neq \frac{7}{9}$ and the numbers are not proportional.</td><td>3. Determine whether 5, 9 and 20, 36 are proportional.
A. Yes
B. No</td></tr>
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Objective [4.1d] Solve proportions.

Brief Procedure	Example	Practice Exercise
To solve $\frac{x}{a} = \frac{c}{d}$, equate cross products and divide on both sides to get x alone.	Solve: $\frac{5}{4} = \frac{y}{11}$. $\frac{5}{4} = \frac{y}{11}$ $5 \cdot 11 = 4 \cdot y$ Equating cross products $\frac{5 \cdot 11}{4} = y$ Dividing by 4 $\frac{55}{4} = y$ The solution is $\frac{55}{4}$.	4. Solve: $\frac{6}{x} = \frac{5}{3}$. A. $\frac{5}{2}$ B. $\frac{18}{5}$ C. 10 D. 12

Objective [4.1e] Solve applied problems involving proportions.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	Louis bought 3 tickets to a campus theater production for \$16.50. How much would 8 tickets cost? 1. *Familiarize.* Let c = the cost of 8 tickets. 2. *Translate.* We translate to a proportion, keeping the number of tickets in the numerator. Tickets $\rightarrow$ $\frac{3}{16.50} = \frac{8}{c}$ $\leftarrow$ Tickets Cost $\rightarrow$ $\leftarrow$ Cost 3. *Solve.* We solve the proportion. $3 \cdot c = 16.50 \cdot 8$ $c = \frac{16.50 \cdot 8}{3}$ $c = 44$ 4. *Check.* We use a different approach as a check. Find the cost per ticket and then multiply it by 8: \$16.50 ÷ 3 = \$5.50 and \$5.50 × 8 = \$44. The answer checks. 5. *State.* Eight tickets would cost \$44.	5. On a map $\frac{1}{2}$ in. represents 40 mi. If two cities are $2\frac{1}{4}$ in. apart on the map, how far apart are they in reality? A. 90 mi B. 100 mi C. 150 mi D. 180 mi

Objective [4.2a] Write three kinds of notation for percent.		
Brief Procedure	Example	Practice Exercise
Using a ratio, write $n\%$ (the ratio of n to 100) as $\frac{n}{100}$. Using fractional notation, write $n\%$ as $n \times \frac{1}{100}$. Using decimal notation, write $n\%$ as $n \times 0.01$.	Write three kinds of notation for 67%. Ratio: $67\% = \frac{67}{100}$ Fractional notation: $67\% = 67 \times \frac{1}{100}$ Decimal notation: $67\% = 67 \times 0.01$	6. Which is not correct notation for 13%? A. 13×0.01 B. $\frac{13}{100}$ C. $13 \times \frac{1}{100}$ D. $\frac{0.13}{100}$
Objective [4.2b] Convert from percent notation to decimal notation.		
Brief Procedure	Example	Practice Exercise
To convert from percent notation to decimal notation, a) replace the percent symbol % with $\times 0.01$, and b) multiply by 0.01, which means move the decimal point two places to the left.	Find decimal notation for 4.3%. a) Replace the percent symbol with $\times 0.01$. 4.3×0.01 b) Move the decimal point two places to the left. 0.04.3 Thus, $4.3\% = 0.043$.	7. Find decimal notation for 54.8%. A. 0.0548 B. 0.548 C. 5.48 D. 548
Objective [4.2c] Convert from decimal notation to percent notation.		
Brief Procedure	Example	Practice Exercise
To convert from decimal notation to percent notation, multiply by 100%. That is, a) move the decimal point two places to the right, and b) write a % symbol.	Find percent notation for 0.09. a) Move the decimal point two places to the right. 0.09. b) Write a percent symbol: 9% Thus, $0.09 = 9\%$.	8. Find percent notation for 1.5. A. 0.015% B. 0.15% C. 15% D. 150%

Objective [4.3a] Convert from fractional notation to percent notation.

Brief Procedure	Example	Practice Exercise
To convert from fractional notation to percent notation, a) find decimal notation by division, and b) convert the decimal notation to percent notation.	Find percent notation for $\frac{3}{5}$. Find decimal notation by division. $5\overline{)3.0}$ (quotient 0.6; 30; 0) $\frac{3}{5} = 0.6$ Convert to percent notation. $0.60. \rightarrow$ $\frac{3}{5} = 60\%$ When the denominator of the fraction is a factor of 100, we can also find percent notation by first multiplying by 1 to get 100 in the denominator: $\frac{3}{5} = \frac{3}{5} \cdot \frac{20}{20} = \frac{60}{100} = 60\%.$	9. Find percent notation for $\frac{7}{8}$. A. 62.5% B. $77.\overline{7}\%$ C. 78% D. 87.5%

Objective [4.3b] Convert from percent notation to fractional notation..

Brief Procedure	Example	Practice Exercise
To convert from percent notation to fractional notation, a) use the definition of percent as a ratio, and b) simplify, if possible.	Find fractional notation for 40%. $40\% = \frac{40}{100} = \frac{20 \cdot 2}{20 \cdot 5} = \frac{20}{20} \cdot \frac{2}{5} = \frac{2}{5}$	10. Find fractional notation for 85%. A. $\frac{11}{20}$ B. $\frac{13}{20}$ C. $\frac{17}{20}$ D. $\frac{19}{20}$

Objective [4.4a] Translate percent problems to equations.

Brief Procedure	Example	Practice Exercise
Keep in mind that "of" translates to "$\cdot$", or "$\times$"; "what" translates to any letter; and "is" translates to '='.	Translate to an equation: What is 12% of 84? $\underbrace{\text{What}}$ is 12% of 84? $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$ $a \quad = 12\% \cdot 84$	11. Translate to an equation: 16% of what is 224? A. $a = 16\% \cdot 224$ B. $16\% \times b = 224$ C. $16 = n \times 224$ D. $16\% = a \times 224$

Objective [4.4b] Solve basic percent problems (using equations).

Brief Procedure	Example	Practice Exercise
Translate to an equation. Then solve the equation.	What percent of 60 is 21? *Translate*: $\underbrace{\text{What percent}}$ of 60 is 21? $\downarrow \quad \downarrow\ \downarrow\ \downarrow\ \downarrow$ $n \quad \times\ 60 =\ 21$ *Solve*: We divide by 60 on both sides and convert the answer to percent notation. $\frac{n \times 60}{60} = \frac{21}{60}$ $n = 0.35 = 35\%$ Thus, 35% of 60 is 21. The answer is 35%.	12. 120% of \$80 is what? A. \$1.50 B. \$66.67 C. \$96 D. \$120

Objective [4.5a] Translate percent problems to proportions.

Brief Procedure	Example	Practice Exercise
Translate as follows: $\frac{\text{Number}}{100} = \frac{\text{Amount}}{\text{Base}}$, or $\frac{N}{100} = \frac{a}{b}$. Keep in mind that the base b usually follows the words "percent of" and N% always translates to $\frac{N}{100}$. We can also read the proportion from a comparison drawing: Percents — Quantities 0% — 0 N% — amount 100% — base	7 is 15% of what? $\downarrow$ $\downarrow$ $\downarrow$ amount number of hundredths base $\frac{15}{100} = \frac{7}{b}$	13. Translate to a proportion: 30 is what percent of 45? A. $\frac{N}{100} = \frac{30}{45}$ B. $\frac{30}{100} = \frac{N}{45}$ C. $\frac{45}{100} = \frac{30}{N}$ D. $\frac{N}{100} = \frac{45}{30}$

<table>
<tr><td colspan="3">Objective [4.5b] Solve basic percent problems (using proportions).</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Translate to a proportion. Then solve the proportion.</td><td>75% of 150 is what?

Translate: $\frac{75}{100} = \frac{a}{150}$

Solve:

$75 \cdot 150 = 100 \cdot a$ Equating cross products
$\frac{75 \cdot 150}{100} = \frac{100 \cdot a}{100}$ Dividing by 100
$\frac{11,250}{100} = a$.
$112.5 = a$

Thus, 75% of 150 is 112.5. The answer is 112.5.</td><td>14. 4% of what is 3.6?
A. 14.4
B. 60.5
C. 90
D. 111</td></tr>
<tr><td colspan="3">Objective [4.6a] Solve applied problems involving percent.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Use the five-step problem solving process.</td><td>On a test of 40 items, James had 34 correct. What percent were correct?

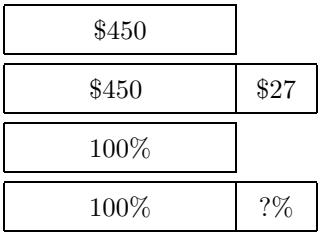
1. Familiarize. The problem asks for a percent. Let n = the percent of test items that were correct.

2. Translate. Rephrase the question and translate.
34 is what percent of 40?
$\downarrow \ \downarrow \quad \downarrow \quad \downarrow \ \downarrow$
$34 = \quad n \quad \times \ 40$

3. Solve. We divide by 40 on both sides and convert the answer to percent notation.
$34 = n \times 40$
$\frac{34}{40} = \frac{n \times 40}{40}$
$0.85 = n$
$85\% = n$

4. Check. We can repeat the calculation. The answer checks.
5. State. 85% of the test items were correct.</td><td>15. The Collins spend 5% of their income on clothing. If their annual income is $33,000, how much is spent on clothing in a year?
A. $165
B. $1650
C. $3300
D. $6600</td></tr>
</table>

Objective [4.6b] Solve applied problems involving percent of increase or decrease.

Brief Procedure	Example	Practice Exercise
To find a percent of increase or decrease: a) Find the amount of increase or decrease. b) Then determine what percent this is of the original amount.	Jo's supervisor tells her that her weekly salary of \$450 will be increased to \$477. What is the percent of increase? 1. *Familiarize.* We find the amount of increase and then make a drawing. $\begin{array}{rl} 4\,7\,7 & \text{New salary} \\ -\,4\,5\,0 & \text{Original salary} \\ \hline 2\,7 & \text{Increase} \end{array}$  2. *Translate.* We rephrase the question and translate. \$27 is what percent of \$450? $\begin{array}{ccccccc} \downarrow & \downarrow & & \downarrow & & \downarrow & \downarrow \\ 27 & = & & n & \times & & 450 \end{array}$ 3. *Solve.* We divide by 450 on both sides and convert the answer to percent notation. $27 = n \times 450$ $\frac{27}{450} = \frac{n \times 450}{450}$ $0.06 = n$ $6\% = n$ 4. *Check.* Note that with a 6% increase, the new salary would be 106% of the original salary. Since 106% of \$450 = $1.06 \times$ \$450 = \$477, the answer checks. 5. *State.* The percent of increase is 6%.	16. During a sale, the price of a jacket decreased from \$120 to \$102. What was the percent of decrease? A. 15% B. 18% C. 20% D. 25%

<table>
<tr><td colspan="3">Objective [4.7a] Solve applied problems involving sales tax and percent.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Keep these facts in mind:
Sales tax = Sales tax rate × Purchase price
Total price = Purchase price + Sales tax</td><td>The sales tax rate in Indiana is 5%. How much tax is charged on a bread machine costing $150? What is the total price?

The sales tax is
$\underbrace{\text{Sales tax rate}}_{\downarrow} \times \underbrace{\text{Purchase price}}_{\downarrow}$
$5\% \quad \times \quad \$150,$

or 0.05×150, or 7.5. Thus, the sales tax is $7.50.

The total price is the purchase price plus the sales tax:
$150 + $7.50, or $157.50.</td><td>17. The sales tax rate is Connecticut is 8%. What is the total price of a digital camera that sells for $550?
A. $44
B. $575
C. $594
D. $615</td></tr>
<tr><td colspan="3">Objective [4.7b] Solve applied problems involving commission and percent.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr><td>Keep this fact in mind:
Commission = Commission rate × Sales</td><td>Celina earns a commission of $1750 selling $25,000 worth of office equipment. What is the commission rate?
$\text{Commission} = \text{Commission rate} \times \text{Sales}$
$1750 = r \times 25,000$
To solve this equation we divide by 25,000 on both sides.
$\frac{1750}{25,000} = \frac{r \cdot 25,000}{25,000}$
$\frac{250 \cdot 7}{250 \cdot 100} = r$
$\frac{250}{250} \cdot \frac{7}{100} = r$
$\frac{7}{100} = r$
$7\% = r$

The commission rate is 7%.</td><td>18. Frank's commission rate is 12%. He receives a commission of $180 on the sale of sporting goods. How much did the sporting goods cost?
A. $1320
B. $1500
C. $4750
D. $6500</td></tr>
</table>

Objective [4.7c] Solve applied problems involving discount and percent.

Brief Procedure	Example	Practice Exercise
Keep these facts in mind: Discount = Rate of discount × Original price Sale price = Original price − Discount	A shirt marked \$45 is on sale at 20% off. What is the discount? the sale price? $\text{Discount} = \text{Rate of discount} \times \text{Original price}$ $D = 20\% \times 45$ We convert 20% to decimal notation and multiply. $\begin{array}{r} 45 \\ \times\ 0.2 \\ \hline 9.0 \end{array}$ The discount is \$9. $\text{Sale price} = \text{Original price} - \text{Discount}$ $S = 45 - 9$ We subtract. $\begin{array}{r} 45 \\ -\ 9 \\ \hline 36 \end{array}$ The sale price is \$36.	19. A sofa marked \$650 is on sale at 30% off. What is the sale price? A. \$195 B. \$375 C. \$455 D. \$515

Objective [4.7d] Solve applied problems involving simple interest and percent.

Brief Procedure	Example	Practice Exercise
The simple interest I on principal P, invested for t years at interest rate r, is given by $I = P \cdot r \cdot t.$	What is the interest on \$4400 invested at an interest rate of 8% for $\frac{1}{2}$ year? Substitute \$4400 for P, 8% for r, and $\frac{1}{2}$ for t in the simple interest formula. $I = P \cdot r \cdot t$ $= \$4400 \times 8\% \times \frac{1}{2}$ $= \frac{\$4400 \times 0.08}{2}$ $= \$176$ The interest for $\frac{1}{2}$ year is \$176.	20. What is the interest on \$1200 invested at 6% for $\frac{1}{4}$ year? A. \$8 B. \$18 C. \$36 D. \$72

Objective [4.7e] Solve applied problems involving compound interest.		
Brief Procedure	Example	Practice Exercise
If a principal P has been invested at interest rate r, compounded n times a year, in t years it will grow to an amount A given by $A = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}$	The Jensens invest \$2000 in an account paying 12%, compounded semiannually. Find the amount in the account after $1\frac{1}{2}$ years. Substitute \$2000 for P, 12% for r, 2 for n, and $1\frac{1}{2}$, or $\frac{3}{2}$, for t in the compound interest formula. $A = P \cdot \left(1 + \frac{r}{n}\right)^{n \cdot t}$ $= \$2000 \times \left(1 + \frac{0.12}{2}\right)^{2 \cdot \frac{3}{2}}$ $= \$2000 \times (1 + 0.06)^3$ ≈ 2382.03 The amount in the account after $1\frac{1}{2}$ years is \$2382.03.	21. The Shaws invest \$3600 in an account paying 8%, compounded quarterly. Find the amount in the account after 1 year. A. \$3745.44 B. \$3880.00 C. \$3893.73 D. \$3896.76