Intermediate Algebra
Chapter R Review

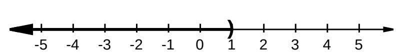
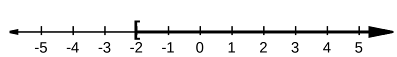
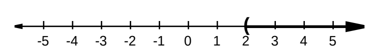
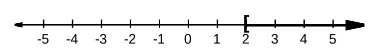
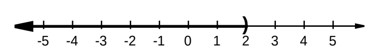
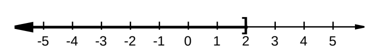
Objective [R.1a] Use roster and set-builder notation to name sets, and distinguish among various kinds of real numbers.

Brief Procedure	Example	Practice Exercises
To use roster notation to name a set, list all the objects in the set.	Use roster notation to name the set of negative integers greater than -5. $\{-4,\ -3,\ -2,\ -1\}$	1. Use roster notation to name the set of positive even integers less than 10. A. $\{2,\ 4,\ 6,\ 8\}$ B. $\{2,\ 4,\ 6,\ 8,\ 10\}$ C. $\{0,\ 2,\ 4,\ 6,\ 8\}$ D. $\{0,\ 2,\ 4,\ 6,\ 8,\ 10\}$
To use set-builder notation to name a set, specify the conditions under which an object is in the set.	Use set-builder notation to name the set of negative integers greater than -5. $\{x \mid x$ is a negative integer greater than $-5\}$ This is read "the set of all x such that x is a negative integer greater than -5."	2. Use set-builder notation to name the set of real numbers less than or equal to 12. A. $\{x \mid x < 12\}$ B. $\{x \mid x \leq 12\}$ C. $\{x \mid x \geq 12\}$ D. $\{x \mid x < -12\}$
To distinguish among various kinds of real numbers, keep the following in mind. Natural numbers: $\{1,\ 2,\ 3,\ \ldots\}$ Whole numbers: $\{0,\ 1,\ 2,\ 3,\ \ldots\}$ Integers: $\{\ldots,\ -2,\ -1,\ 0,\ 1,\ 2,\ \ldots\}$ Rational numbers: $\left\{\frac{p}{q} \middle\vert p \text{ and } q \text{ are integers and } q \neq 0\right\}$ Irrational numbers: Numbers whose decimal representation neither terminates nor repeats. They cannot be represented as the quotient of two integers. Real numbers: $\{x \mid x$ is a rational number or x is an irrational number$\}$	Given the numbers -5, $\frac{7}{12}$, 2.68, 14, 0, $\sqrt{18}$, $0.212112111\ldots$, and $\sqrt{64}$, name the integers. The numbers in the set of integers are -5, 14, 0, and $\sqrt{64}$. (We include $\sqrt{64}$ because $\sqrt{64} = 8$.)	3. Given the numbers in the example at the left, name the rational numbers. A. -5, $\frac{7}{12}$, 2.68, 14, 0, $\sqrt{64}$ B. -5, $\frac{7}{12}$, 2.68, 14, 0, $\sqrt{18}$, $\sqrt{64}$ C. -5, $\frac{7}{12}$, 2.68, 14, 0, $0.212112111\ldots$, $\sqrt{64}$ D. All of them

Objective [R.1b] Determine which of two real numbers is greater and indicate which, using $<$ or $>$; given an inequality like $a < b$, write another inequality with the same meaning; and determine whether an inequality like $-2 \leq 3$ or $4 > 5$ is true.

Brief Procedure	Example	Practice Exercises
To determine which of two real numbers is greater, consider the relative position of the two numbers on the number line. The one on the left is less than the one on the right. The symbol $<$ means "is less than" and the symbol $>$ means "is greater than."	Use $<$ or $>$ for $\square$ to write a true sentence: $-7 \ \square \ -10$ Since -7 is to the right of -10 on the number line, we have $-7 > -10$.	4. Use $<$ or $>$ for $\square$ to write a true sentence: $-8 \ \square \ 1$ A. $<$ B. $>$
Given an inequality like $a < b$, write another inequality with the same meaning by interchanging a and b and reversing the direction of the inequality symbol.	Write another inequality with the same meaning as $x > 8$. The inequality $8 < x$ has the same meaning.	5. Write another inequality with the same meaning as $-3 < t$. A. $t < -3$ B. $t > 3$ C. $3 < t$ D. $t > -3$
An inequality like $-2 \leq 3$ is true if either $-2 < 3$ is true or $-2 = 3$ is true. If neither is true, then the inequality is false. To determine whether an inequality like $4 > 5$ is true, determine the relative positions of the numbers on the number line. The one farther to the right is larger.	Determine whether each inequality is true or false. a) $-4 \leq 1$ b) $6 \geq 6$ c) $-10 \geq 2$ a) $-4 \leq 1$ is true since $-4 < 1$ is true. b) $6 \geq 6$ is true since $6 = 6$ is true. c) $-10 \geq 2$ is false since neither $-10 > 2$ nor $-10 = 2$ is true.	6. Determine whether the inequality $-1 \geq -8$ is true or false. A. True B. False

Objective [R.1c] Graph inequalities on the number line.

Brief Procedure	Example	Practice Exercise
Shade all points on the number line that are solutions of the given inequality.	Graph each inequality. a) $x < 1$ b) $x \geq -2$ a) The solutions of $x < 1$ are all numbers less than 1. We shade all points to the left of 1. Use an open circle at 1 to indicate that 1 is not part of the graph.  b) The solutions of $x \geq -2$ are all numbers greater than -2 and the number -2 as well. We shade all points to the right of -2, and we use a closed circle at -2 to indicate that -2 is part of the graph.  	7. Graph $x > 2$. A.  B.  C.  D. 

Objective [R.1d] Find the absolute value of a real number.		
Brief Procedure	Example	Practice Exercise
If the number is negative, make it positive. If the number is positive or zero, leave it alone.	Find $\|-4.3\|$. The number is negative, so we make it positive. $\|-4.3\| = 4.3$	8. Find $\|59\|$. A. -59 B. 0 C. 59

Objective [R.2a] Add real numbers.		
Brief Procedure	Example	Practice Exercise
1. *Positive numbers*: Add the same as arithmetic numbers. The answer is positive. 2. *Negative numbers*: Add absolute values. The answer is negative. 3. *A positive and a negative number*: Subtract the smaller absolute value from the larger. Then: a) If the positive number has the greater absolute value, the answer is positive. b) If the negative number has the greater absolute value, the answer is negative. c) If the numbers have the same absolute value, the answer is 0. 4. *One number is zero*: The sum is the other number.	Add: $-15 + 9$. We have a negative and a positive number. The absolute values are 15 and 9. The difference is 6. The negative number has the larger absolute value, so the answer is negative. $-15 + 9 = -6$	9. Add: $-1.2 + (-3.4)$. A. 4.6 B. 2.2 C. -2.2 D. -4.6

Objective [R.2b] Find the opposite, or additive inverse, of a real number.		
Brief Procedure	Example	Practice Exercise
The opposite, or additive inverse, of any real number a is the number $-a$ such that $a + (-a) = (-a) + a = 0$. To find the opposite of a number, we change its sign.	Find the opposite of $\frac{5}{3}$. The opposite of $\frac{5}{3}$ is $-\frac{5}{3}$ because $\frac{5}{3} + \left(-\frac{5}{3}\right) = 0$.	10. Find the opposite of -20. A. -20 B. 0 C. 20

Objective [R.2c] Subtract real numbers.

Brief Procedure	Example	Practice Exercise
For any real numbers a and b, $a - b = a + (-b)$. (To subtract, add the opposite, or additive inverse, of the number being subtracted.)	Subtract: $6 - (-7)$. The opposite of -7 is 7. We change the subtraction to addition and add the opposite. $6 - (-7) = 6 + 7 = 13$	11. Subtract: $2 - 12$. A. -14 B. -10 C. 10 D. 14

Objective [R.2d] Multiply real numbers.

Brief Procedure	Example	Practice Exercise
a) Multiply the absolute values. b) If the signs are the same, the answer is positive. c) If the signs are different, the answer is negative.	Multiply: $-2.4(3)$. The signs are different, so the answer is negative. $-2.4(3) = -7.2$	12. Multiply: $-7(-9)$. A. -63 B. -16 C. 2 D. 63

Objective [R.2e] Divide real numbers.

Brief Procedure	Example	Practice Exercise
For any real numbers a and b, $b \neq 0$, $a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$. (To divide, we can multiply by the reciprocal of the divisor.)	Divide: $-\frac{1}{3} \div \frac{2}{7}$. $-\frac{1}{3} \div \frac{2}{7} = -\frac{1}{3} \cdot \frac{7}{2} = -\frac{7}{6}$	13. Divide: $-\frac{3}{4} \div \left(-\frac{5}{11}\right)$. A. $-\frac{53}{44}$ B. $-\frac{13}{44}$ C. $\frac{15}{44}$ D. $\frac{33}{20}$

Objective [R.3a] Rewrite expressions with whole-number exponents, and evaluate exponential expressions.

Brief Procedure	Example	Practice Exercises
To rewrite expressions with whole-number exponents, count the number of identical factors. Then make that number the exponent, using the repeated factor as the base.	Write exponential notation for $6 \cdot 6 \cdot 6 \cdot 6$. $\underbrace{6 \cdot 6 \cdot 6 \cdot 6}_{\text{4 factors}} = 6^4$	14. Write exponential notation for $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. A. 32 B. $5 \cdot 2$ C. 5^2 D. 2^5
To evaluate exponential expressions, rewrite the exponential expression as a product and compute.	Evaluate: 3^4. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$	15. Evaluate: 5^3. A. 15 B. 125 C. 243 D. 625

Objective [R.3b] Rewrite expressions with or without negative integers as exponents.		
Brief Procedure	**Example**	**Practice Exercises**
For any real number a that is nonzero and any integer n, $\frac{1}{a^n} = a^{-n}$.	Rewrite using positive exponents. a) $3x^{-8}$ b) $\frac{1}{y^{-2}}$ a) $3x^{-8} = 3 \cdot \frac{1}{x^8} = \frac{3}{x^8}$ b) $\frac{1}{y^{-2}} = y^{-(-2)} = y^2$	16. Rewrite $2n^{-5}$ using a positive exponent. A. $\frac{1}{2n^5}$ B. $\frac{2}{n^5}$ C. $\frac{n^5}{2}$ D. $2n^5$
For any real number a that is nonzero and any integer n, $\frac{1}{a^n} = a^{-n}$.	Rewrite $\frac{1}{x^3}$ using a negative exponent. $\frac{1}{x^3} = x^{-3}$	17. Express $\frac{1}{5^4}$ using a negative exponent. A. $\frac{1}{5^{-4}}$ B. $\frac{1}{(-5)^4}$ C. 5^4 D. 5^{-4}

Objective [R.3c] Simplify expressions using the rules for order of operations.		
Brief Procedure	**Example**	**Practice Exercise**
1. Do all calculations within grouping symbols before operations outside. 2. Evaluate all exponential expressions. 3. Do all multiplications and divisions in order from left to right. 4. Do all additions and subtractions in order from left to right.	Simplify: $64 \div 4^2 \cdot 3 + (12 - 7)$. $64 \div 4^2 \cdot 3 + (12 - 7)$ $= 64 \div 4^2 \cdot 3 + 5$ $= 64 \div 16 \cdot 3 + 5$ $= 4 \cdot 3 + 5$ $= 12 + 5$ $= 17$	18. Simplify: $9 + (19 - 9)^2 \div 5 \cdot 2$. A. 19 B. 49 C. 121 D. 220

Objective [R.4a] Translate a phrase to an algebraic expression.		
Brief Procedure	**Example**	**Practice Exercise**
Learn which words translate to certain operation symbols. (See page 33 in the text.) Choose a variable or variables to correspond to the number or numbers involved. It can be helpful to try some numerical examples before writing the algebraic expression.	Translate to an algebraic expression: Four less than some number. Let n = the number. Now if the number were 7, then the translation would be $7 - 4$. Similarly, if the number were 52, then the translation would be $52 - 4$. Thus, we see from these numerical examples, that if the number were n, the translation would be $n - 4$.	19. Translate to an algebraic expression: Three times some number. A. $n + 3$ B. $n - 3$ C. $3 - n$ D. $3n$

Objective [R.4b] Evaluate an algebraic expression by substitution.

Brief Procedure	Example	Practice Exercise
Substitute for the variable(s) and carry out the resulting calculation.	Evaluate $m - n$ for $m = 29$ and $n = 12$. Substitute 29 for m and 12 for n and carry out the subtraction. $m - n = 29 - 12 = 17$	20. Evaluate $\frac{x}{y}$ for $x = 72$ and $y = 9$. A. $\frac{1}{8}$ B. 8 C. 63 D. 81

Objective [R.5a] Determine whether two expressions are equivalent by completing a table of values.

Brief Procedure	Example	Practice Exercise
If two expressions have the same value for all allowable replacements, they are equivalent. To determine if this is the case, we can evaluate each expression for some values of the variable and organize the results in a table.	Complete the table by evaluating each expression for the given values. Then determine whether the expressions are equivalent. $\begin{array}{|c|c|c|} \hline & x+x & 2x \\ \hline x=-3 & & \\ \hline x=0 & & \\ \hline x=4 & & \\ \hline \end{array}$ We substitute and find the value of each expression. $\begin{array}{|c|c|c|} \hline & x+x & 2x \\ \hline x=-3 & -6 & -6 \\ \hline x=0 & 0 & 0 \\ \hline x=4 & 8 & 8 \\ \hline \end{array}$ It appears that the expressions have the same value for all allowable replacements, so they are equivalent.	21. Complete the table by evaluating each expression for the given values. Then determine whether the expressions are equivalent. $\begin{array}{|c|c|c|} \hline & 2(x-1) & 2x-1 \\ \hline x=-2 & & \\ \hline x=0 & & \\ \hline x=3 & & \\ \hline \end{array}$ A. Equivalent B. Not equivalent

Objective [R.5b] Find equivalent fractional expressions by multiplying by 1, and simplify fractional expressions.

Brief Procedure	Example	Practice Exercise
To find an equivalent fractional expression, multiply the fraction by 1 using n/n. If a specific denominator is desired, choose n by determining the number the original denominator should be multiplied by in order to get the desired denominator.	Find a name for $\frac{2}{3}$ with a denominator of 12. Since $3 \cdot 4 = 12$, we multiply by $\frac{4}{4}$: $\frac{2}{3} = \frac{2}{3} \cdot \frac{4}{4} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12}$	22. Find a name for $\frac{3}{4}$ with a denominator of 20. A. $\frac{3}{20}$ B. $\frac{8}{20}$ C. $\frac{15}{20}$ D. $\frac{19}{20}$

Objective [R.5b] (continued)		
Brief Procedure	Example	Practice Exercise
To simplify a fractional expression, remove a factor of 1 to get the name for the fraction that has the smallest numerator and denominator.	Simplify: $\frac{16}{36}$. $\frac{16}{36} = \frac{4 \cdot 4}{4 \cdot 9} = \frac{4}{4} \cdot \frac{4}{9} = 1 \cdot \frac{4}{9} = \frac{4}{9}$	23. Simplify: $\frac{9}{24}$. A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{9}{8}$

Objective [R.5c] Use the commutative and associative laws to find equivalent expressions.		
Brief Procedure	Example	Practice Exercises
The Commutative Laws *Addition* For any numbers a and b, $a + b = b + a$. *Multiplication* For any numbers a and b, $ab = ba$. (We can change the order when adding or when multiplying without affecting the result.)	Use a commutative law to write an equivalent expression. a) $n + 6$ b) xy a) An equivalent expression is $6 + n$, by the commutative law of addition. b) An equivalent expression is yx, by the commutative law of multiplication.	24. Use a commutative law to write an equivalent expression for $8 + a$. A. $a + 8$ B. $8a$ C. $a8$ D. $8 - a$
The Associative Laws *Addition* For any numbers a, b, and c, $a + (b + c) = (a + b) + c$. *Multiplication* For any numbers a, b, and c, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. (Numbers can be grouped in any manner for addition and for multiplication.)	Use an associative law to write an equivalent expression. a) $(m + n) + 1$ b) $5(st)$ a) An equivalent expression is $m + (n + 1)$, by the associative law of addition. b) An equivalent expression is $(5s)t$, by the associative law of multiplication.	25. Use an associative law to write an equivalent expression for $(4x)y$. A. $y(4x)$ B. $(x4)y$ C. $4(xy)$ D. $y + (4x)$

Objective [R.5d] Use the distributive laws to find equivalent expressions by multiplying and factoring.		
Brief Procedure	Example	Practice Exercises
To multiply, use the following: For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.	Multiply: $5(2x - 3y + z)$. $5(2x - 3y + z)$ $= 5 \cdot 2x - 5 \cdot 3y + 5 \cdot z$ $= 10x - 15y + 5z$	26. Multiply: $3(x + 4y - 2z)$. A. $3x + 4y - 2z$ B. $3x + 12y + 6z$ C. $3x + 12y - 6z$ D. $3x - 12y - 6z$
To factor, find the largest factor that is common to all the terms of the expression and factor it out.	Factor: $8a + 4b - 12c$. $8a + 4b - 12c$ $= 4 \cdot 2a + 4 \cdot b - 4 \cdot 3c$ $= 4(2a + b - 3c)$	27. Factor: $36m - 27n + 9p$. A. $3(12m - 9n + 3p)$ B. $36(m - 27n + 9p)$ C. $9(4m - 3n)$ D. $9(4m - 3n + p)$

Objective [R.6a] Simplify expressions by collecting like terms.		
Brief Procedure	Example	Practice Exercise
Identify the terms with exactly the same variable, use the distributive laws to factor out the variable, and then simplify.	Collect like terms: $3x - 5y + 8x + y$. $3x - 5y + 8x + y$ $= 3x + 8x - 5y + y$ $= 3x + 8x - 5y + 1 \cdot y$ $= (3 + 8)x + (-5 + 1)y$ $= 11x - 4y$	28. Collect like terms: $6a - 4b - a + 2b$. A. $5a - 2b$ B. $2a + b$ C. $6a - 2b$ D. $5a + 6b$

Objective [R.6b] Simplify an expression by removing parentheses and collecting like terms.		
Brief Procedure	Example	Practice Exercise
Use a distributive law to remove parentheses and then collect like terms.	Remove parentheses and simplify: $6x - 2(x - 3y)$. $6x-2(x-3y) = 6x-2x+6y = 4x+6y$	29. Remove parentheses and simplify: $3m - n - (2m + 5n)$. A. $m + 4n$ B. $5m + 4n$ C. $m - 4n$ D. $m - 6n$

Objective [R.7a] Use exponential notation in multiplication and division.		
Brief Procedure	Example	Practice Exercises
For any number a and any positive integers m and n, $a^m \cdot a^n = a^{m+n}$. (When multiplying with exponential notation, if the bases are the same, keep the base and add the exponents.)	Multiply and simplify: $y^2 \cdot y^6$. $y^2 \cdot y^6 = y^{2+6} = y^8$	30. Multiply and simplify: $x^3 \cdot x^4$. A. x^7 B. $2x^7$ C. x^{12} D. x^{14}
For any nonzero number a and any positive integers m and n, $\frac{a^m}{a^n} = a^{m-n}$. (When dividing with exponential notation, if the bases are the same, keep the base and subtract the exponent of the denominator from the exponent of the numerator.)	Divide and simplify: $\frac{a^{10}b^4}{a^2b}$. $\frac{a^{10}b^4}{a^2b} = \frac{a^{10}}{a^2} \cdot \frac{b^4}{b}$ $= a^{10-2}b^{4-1}$ $= a^8b^3$	31. Divide and simplify: $\frac{x^3y^7}{x^2y^4}$. A. y^3 B. xy^3 C. x^5y^{11} D. x^6y^{28}

Objective [R.7b] Use exponential notation in raising a power to a power and in raising a product or a quotient to a power.

Brief Procedure	Example	Practice Exercises
To raise a power to a power, multiply the exponents. That is, for any real number a and any integers m and n, $(a^m)^n = a^{mn}$.	Simplify: $(y^{-3})^2$. $(y^{-3})^2 = y^{-3\cdot 2} = y^{-6} = \frac{1}{y^6}$	32. Simplify: $(b^{-4})^{-3}$. A. $\frac{1}{b}$ B. $\frac{1}{b^7}$ C. b^7 D. b^{12}
To raise a product to the nth power, raise each factor to the nth power. That is, for any real numbers a and b and any integer n, $(ab)^n = a^n b^n$.	Simplify: $(3x^{-4}y^2)^3$. $(3x^{-4}y^2)^3 = 3^3(x^{-4})^3(y^2)^3$ $= 27x^{-12}y^6$ $= \frac{27y^6}{x^{12}}$	33. Simplify: $(8a^3b^{-5})^2$. A. $\frac{8a^5}{b^7}$ B. $\frac{16a^6}{b^{10}}$ C. $\frac{64a^3}{b^5}$ D. $\frac{64a^6}{b^{10}}$
To raise a quotient to a power, raise both the numerator and the denominator to the power. That is, for any real numbers a and b, $b \neq 0$, and any integer n, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.	Simplify: $\left(\frac{4}{a^5}\right)^3$. $\left(\frac{4}{a^5}\right)^3 = \frac{4^3}{(a^5)^3} = \frac{64}{a^{15}}$	34. Simplify: $\left(\frac{y^4}{7}\right)^2$. A. $\frac{y^6}{49}$ B. $\frac{y^8}{49}$ C. $\frac{y^{16}}{49}$ D. $\frac{y^8}{7}$

Objective [R.7c] Convert between decimal notation and scientific notation and use scientific notation with multiplication and division.

Brief Procedure	Example	Practice Exercises
To convert from decimal notation to scientific notation, rewrite the number in the form $M \times 10^n$, where n is an integer, $1 \leq M < 10$, and M is expressed in decimal notation. If the original number is large (greater than 1), then n is positive. If it is a small number (less than 1), then n is negative.	Convert 0.00048 to scientific notation. 0.0004. 8 └──↑ 4 places The number is small, so the exponent is negative. $0.00048 = 4.8 \times 10^{-4}$	35. Convert 567,000 to scientific notation. A. 5.67×10^{-5} B. 5.67×10^3 C. 5.67×10^5 D. 567×10^3

Objective [R.7c] (continued)		
Brief Procedure	Example	Practice Exercises
Given a number $M \times 10^n$ in scientific notation, convert to decimal notation by moving the decimal point in M n places to the right or left. If the exponent is positive, the number is large, so the decimal point should be moved to the right. If the exponent is negative, the number is small so the decimal point should be moved to the left.	Convert 4.208×10^6 to decimal notation. The exponent is positive, so the number is large. We move the decimal point 6 places to the right. 4.208000. └──────↑ 6 places $4.208 \times 10^6 = 4,208,000$	36. Convert 3×10^{-4} to decimal notation. A. 0.0003 B. 0.003 C. 3000 D. 30,000
To use scientific notation with multiplication and division, apply the commutative and associative laws and the rules for exponents.	Multiply and express the result in scientific notation: $(4.2 \times 10^8) \cdot (3.1 \times 10^{-3})$. $(4.2 \times 10^8) \cdot (3.1 \times 10^{-3})$ $= (4.2 \cdot 3.1) \times (10^8 \cdot 10^{-3})$ $= 13.02 \times 10^5$ The answer at this stage is 13.02×10^5, but this is not scientific notation, because 13.02 is not a number between 1 and 10. We convert 13.02 to scientific notation and simplify. 13.02×10^5 $= (1.302 \times 10) \times 10^5$ $= 1.302 \times (10 \times 10^5)$ $= 1.302 \times 10^6$	37. Divide and express the result in scientific notation: $\dfrac{3.3 \times 10^2}{4.4 \times 10^{-10}}$. A. 0.75×10^{-8} B. 0.75×10^{12} C. 7.5×10^{11} D. 7.5×10^{13}