Intermediate Algebra
Chapter 4 Review

Objective [4.1a] Identify the degree of each term and the degree of a polynomial; identify terms, coefficients, monomials, binomials, and trinomials; arrange polynomials in ascending or descending order; and identify the leading coefficient.

Brief Procedure	Example	Practice Exercises
The degree of a term is the sum of the exponents of the variables. The degree of a polynomial is the same as the degree of its term of highest degree.	Identify the degree of each term and the degree of the polynomial: $8x^3 - 15x^2y^3 + 4xy + y^4 - 7$. The degree of $8x^3$ is the exponent of the variable, 3. The degree of $-15x^2y^3$ is the sum of the exponents of the variables, $2+3$, or 5. The degree of $4xy$ is the sum of the exponents of the variables, $1+1$, or 2. The degree of y^4 is the exponent of the variable, 4. The degree of -7 is the exponent of the variable, 0, since $-7 = -7x^0$. The degree of the term of highest degree is 5, so the degree of the polynomial is 5.	1. Identify the degree of the polynomial: $6a^2b - 4ab + 2b^4 + 10$. A. 2 B. 3 C. 4 D. 10
To identify the terms of a polynomial, rewrite the subtractions in the polynomial as additions. Then each expression being added is a term of the polynomial.	Identify the terms of the polynomial $3y^3 - 2y^2 - 5y + 1$. $3y^3 - 2y^2 - 5y + 1 =$ $3y^3 + (-2y^2) + (-5y) + 1$ Then the terms are $3y^3,\ -2y^2,\ -5y$, and 1.	2. Identify the terms of the polynomial $-5y^4 + 3y^2 - 2$. A. $5y^4,\ 3y^2,\ 2$ B. $5y^4,\ 3y^2$ C. $-5y^4,\ 3y^2$ D. $-5y^4,\ 3y^2,\ -2$
The coefficient of a term of a polynomial is the number by which the variable is multiplied.	Identify the coefficients of each term of the polynomial $5y^6 - 10y^2 + 4$. The coefficient of $5y^6$ is 5. The coefficient of $-10y^2$ is -10. The coefficient of 4 is 4.	3. Identify the coefficients of each term of the polynomial $-8x^3 + 4x^2 - 7$. A. $-8,\ 4$ B. $-8,\ 4,\ -7$ C. 3, 2 D. 3, 2, 0
A polynomial with just one term is a monomial. A polynomial with just two terms is a binomial. A polynomial with just three terms is a trinomial. Those with more than three terms do not generally have a specific name.	Classify each of the following as a monomial, binomial, trinomial, or none of these. a) $x^2 - 7$ b) $2x^3 - x^2 + 5x + 6$ a) This polynomial has just two terms, so it is a binomial. b) This polynomial has more than three terms, so it is none of these.	4. Classify $-6x^7$ as a monomial, binomial, trinomial, or none of these. A. Monomial B. Binomial C. Trinomial D. None of these

Objective [4.1a] (continued)		
Brief Procedure	Example	Practice Exercises
To arrange a polynomial in ascending order, write the terms so that the exponents increase from left to right. To arrange a polynomial in descending order, write the terms so that the exponents decrease from left to right.	Arrange $3t-4t^2-7+2t^3$ in ascending order and then in descending order. Ascending order: $-7+3t-4t^2+2t^3$ Descending order: $2t^3-4t^2+3t-7$	5. Arrange $8-3x^2+4x^3-x$ in descending order. A. $8-x-3x^2+4x^3$ B. $-x-3x^2+4x^3+8$ C. $-3x^2-x+4x^3+8$ D. $4x^3-3x^2-x+8$
The leading coefficient of a polynomial is the coefficient of the term of highest degree.	Identify the leading coefficient of $5x^3-6x^2y^2-4xy^4+1$. The degrees of the terms are 3, 4, 5, and 0, respectively. Thus, the term of highest degree is $-4xy^4$, so the leading coefficient is -4.	6. Identify the leading coefficient of $12s^2t+3s^2t^2-4st-3$. A. 12 B. 3 C. -4 D. -3

Objective [4.1b] Evaluate a polynomial function for given inputs.		
Brief Procedure	Example	Practice Exercise
Substitute the given input for each occurrence of the variable and carry out the resulting calculations.	For the polynomial function $P(x)=x^2-4x+5$, find $P(-1)$. $P(-1)=(-1)^2-4(-1)+5$ $=1+4+5=10$	7. For the polynomial function $P(x)=2x^2+x-8$, find $P(2)$. A. -5 B. -2 C. 2 D. 5

Objective [4.1c] Collect like terms in a polynomial and add polynomials.		
Brief Procedure	Example	Practice Exercises
Terms that have the same variable(s) raised to the same power(s) are like terms. The distributive laws allow us to collect like terms by adding or subtracting their coefficients.	Collect like terms: $5x^4-6x^2y-3x^4+y$. $5x^4-6x^2y-3x^4+y$ $=(5-3)x^4-6x^2y+y$ $=2x^4-6x^2y+y$	8. Collect like terms: $4x^3-2x^2+3x^2-5$. A. $7x^3-2x^2-5$ B. $4x^3+x^2-5$ C. $5x^2-5$ D. x^2
To add two polynomials, write a plus sign between them and then collect like terms. The polynomials can also be written with like terms in columns and then added.	Add: $(5x^3+x-7)+(2x^3-4x^2+3)$. $(5x^3+x-7)+(2x^3-4x^2+3)$ $=(5+2)x^3-4x^2+x+(-7+3)$ $=7x^3-4x^2+x-4$	9. Add: $(6x^4y^2-5x^2y-1)+(x^4y^2+2x^2+3)$. A. $6x^4y^2-4x^2y+2x^2+2$ B. $7x^4y^2-3x^2+2$ C. $7x^4y^2-3x^2y+2$ D. $7x^4y^2-5x^2y+2x^2+2$

Objective [4.1d] Find the opposite of a polynomial and subtract polynomials.

Brief Procedure	Example	Practice Exercises
The opposite of a polynomial P can be symbolized $-P$ or by replacing each term with its opposite.	Write two equivalent expressions for the opposite of $2xy - 3xy^2 + y - 5$. One expression is $-(2xy - 3xy^2 + y - 5)$. A second expression is $-2xy + 3xy^2 - y + 5$.	10. Which is not an expression for the opposite of $x^2 - xy^3 + 4y$? A. $-(x^2 - xy^3 + 4y)$ B. $-x^2 - xy^3 + 4y$ C. $-x^2 + xy^3 - 4y$
To subtract polynomials, add the opposite of the polynomial being subtracted. Some steps can be skipped if we mentally take the opposite of each term being subtracted and then combine like terms.	Subtract: $(4x^2 - x + 3) - (6x^2 - 4x - 1)$. $(4x^2 - x + 3) - (6x^2 - 4x - 1)$ $= 4x^2 - x + 3 - 6x^2 + 4x + 1)$ $= -2x^2 + 3x + 4$	11. Subtract: $(x^3 - x + 2) - (5x^3 + x^2 - 8)$. A. $-4x^3 + 10$ B. $-4x^3 + x^2 - x - 6$ C. $-4x^3 - x^2 - x - 6$ D. $-4x^3 - x^2 - x + 10$

Objective [4.2a] Multiply any two polynomials.

Brief Procedure	Example	Practice Exercise
Multiply each term of one polynomial by every term of the other and collect like terms, if possible. It is sometimes convenient to write the multiplication in columns.	Multiply. a) $(2x - 7)(x + 3)$ b) $(x^2 - 2x + 3)(x - 1)$ *a)* $(2x - 7)(x + 3)$ $= (2x - 7)x + (2x - 7)3$ $= 2x \cdot x - 7 \cdot x + 2x \cdot 3 - 7 \cdot 3$ $= 2x^2 - 7x + 6x - 21$ $= 2x^2 - x - 21$ b) We use columns. First we multiply the top row by -1 and then by x, placing like terms of the product in the same column. Finally we collect like terms. $\begin{array}{rrrr} & x^2 & -\ 2x & +\ 3 \\ & & x & -\ 1 \\ \hline & -\ x^2 & +\ 2x & -\ 3 \\ x^3 & -\ 2x^2 & +\ 3x & \\ \hline x^3 & -\ 3x^2 & +\ 5x & -\ 3 \end{array}$	12. Multiply: $(x^3 - 3x + 1)(x^2 + 4)$. A. $x^5 - 3x^3 + x^2$ B. $x^5 - 12x + 1$ C. $x^5 + x^3 - 11x^2 + 4$ D. $x^5 + x^3 + x^2 - 12x + 4$

Objective [4.2b] Use the FOIL method to multiply two binomials.		
Brief Procedure	Example	Practice Exercise
To multiply two binomials, $A+B$ and $C+D$, multiply the **F**irst terms AC, the **O**utside terms AD, the **I**nside terms BC, and then the **L**ast terms BD. Then collect like terms, if possible. $(A+B)(C+D) = AC+AD+BC+BD$	Multiply: $(2x+3y)(x-4y)$. $(2x+3y)(x-4y)$ F O I L $= 2x\cdot x + 2x\cdot(-4y) + 3y\cdot x + 3y\cdot(-4y)$ $= 2x^2 - 8xy + 3xy - 12y^2$ $= 2x^2 - 5xy - 12y^2$	13. Multiply: $(y+2)(3y-5)$. A. $3y^2+y-10$ B. $3y^2-5y-10$ C. $3y^2+6y-10$ D. $3y^2-11y-10$

Objective [4.2c] Use a rule to square a binomial.		
Brief Procedure	Example	Practice Exercise
The square of a binomial is the square of the first term, plus or minus twice the product of the two terms, plus the square of the last term: $(A+B)^2 = A^2+2AB+B^2$; $(A-B)^2 = A^2-2AB+B^2$.	Multiply: $(3x-4)^2$. $(3x-4)^2$ $= (3x)^2 - 2\cdot 3x\cdot 4 + 4^2$ $= 9x^2 - 24x + 16$	14. Multiply: $(2x+y)^2$. A. $4x^2+y^2$ B. $2x^2+4xy+y^2$ C. $4x^2+2xy+y^2$ D. $4x^2+4xy+y^2$

Objective [4.2d] Use a rule to multiply a sum and a difference of the same two terms.		
Brief Procedure	Example	Practice Exercise
The product of the sum and the difference of the same two terms is the square of the first term minus the square of the second term: $(A+B)(A-B) = A^2-B^2$.	Multiply: $(x+3y)(x-3y)$. $(x+3y)(x-3y) = x^2-(3y)^2$ $= x^2 - 9y^2$	15. Multiply: $(x+6)(x-6)$. A. $x^2+12x-36$ B. $x^2-12x-36$ C. x^2+36 D. x^2-36

Objective [4.2e] For functions f described by second-degree polynomials, find and simplify notation like $f(a+h)$ and $f(a+h)-f(a)$.		
Brief Procedure	Example	Practice Exercise
Substitute the given input(s) and carry out the resulting computations.	Given $f(x) = x^2 - x + 2$, find and simplify $f(a+4)$ and $f(a+h)-f(a)$. $f(a+4)$ $= (a+4)^2 - (a+4) + 2$ $= a^2 + 8a + 16 - a - 4 + 2$ $= a^2 + 7a + 14$ $f(a+h) - f(a)$ $= [(a+h)^2 - (a+h) + 2]$ $-(a^2 - a + 2)$ $= a^2 + 2ah + h^2 - a - h +$ $2 - a^2 + a - 2$ $= 2ah + h^2 - h$	16. Given $f(x) = x^2+2x-1$, find and simplify $f(a+h)-f(a)$. A. $2ah+h^2-2h$ B. $2ah+h^2+2h$ C. $2ah+h^2+4a+2h-2$ D. $2a^2+2ah+h^2+4a+2h-2$

Objective [4.3a] Factor polynomials whose terms have a common factor.		
Brief Procedure	Example	Practice Exercise
Find the largest factor common to all the terms of the polynomial. Then use the distributive law to express the polynomial as a product where one factor is the largest common factor.	Factor $18x^5 - 9x^3 + 27x^2$. Although there are many factors common to the three terms, the *largest* common factor is $9x^2$. $18x^5 - 9x^3 + 27x^2$ $= (9x^2)(2x^3) - (9x^2)(x) + (9x^2)(3)$ $= 9x^2(2x^3 - x + 3)$	17. Factor $24y^8 + 16y^6 - 8y^4$, factoring out the largest common factor. A. $2y(12y^7 + 8y^5 - 4y^3)$ B. $4y^3(6y^5 + 4y^3 - 2y)$ C. $8y^4(3y^4 + 2y^2)$ D. $8y^4(3y^4 + 2y^2 - 1)$

Objective [4.3b] Factor certain polynomials with four terms by grouping.		
Brief Procedure	Example	Practice Exercise
Group the terms into two pairs. Factor each group and then factor the common factor out of the resulting expression.	Factor $6x^3+9x^2-8x-12$ by grouping. $6x^3 + 9x^2 - 8x - 12$ $= (6x^3 + 9x^2) + (-8x - 12)$ $= 3x^2(2x + 3) - 4(2x + 3)$ $= (3x^2 - 4)(2x + 3)$	18. Factor $x^3 - 3x^2 + 2x - 6$ by grouping. A. One factor is $(x + 2)$. B. One factor is $(x^2 + 2)$. C. One factor is $(x^2 - 3)$. D. One factor is $(x + 3)$.

Objective [4.3c] Factor trinomials of the type $x^2 + bx + c$.		
Brief Procedure	Example	Practice Exercise
1. First arrange in descending order. 2. Use a trial-and-error procedure that looks for factors of c whose sum is b. • If c is positive, the signs of the factors are the same as the sign of b. • If c is negative, then one factor is positive and the other is negative. (If the sum of two factors is the opposite of b, changing the sign of each factor will give the desired factors whose sum is b.) 3. Check your result by multiplying.	Factor $x^2 - 2x - 15$. Since the constant term, -15, is negative, we look for a factorization of -15 in which one factor is positive and one factor is negative. The sum of the factors must be the coefficient of the middle term, -2, so the negative factor must have the larger absolute value. The possible pairs of factors that meet these criteria are 1, -15 and 3, -5. The numbers we need are 3 and -5. $x^2 - 2x - 15 = (x + 3)(x - 5)$.	19. Factor $x^2 - 9x + 8$. A. One factor is $(x + 1)$. B. One factor is $(x - 1)$. C. One factor is $(x + 8)$. D. One factor is $(x - 4)$.

Objective [4.4a] Factor trinomials of the type $ax^2 + bx + c, a \neq 1$, by the grouping method.		
Brief Procedure	Example	Practice Exercise
1. Factor out the largest common factor. 2. Multiply the leading coefficient a and the constant c. 3. Try to factor the product ac so that the sum of the factors is b. That is, find integers p and q such that $pq = ac$ and $p + q = b$. 4. Split the middle term. That is, write it as a sum using the factors found in step (3). 5. Then factor by grouping.	Factor $5x^2 + 7x - 6$ by grouping. 1. There is no common factor (other than 1 or -1). 2. Multiply the leading coefficient 5 and the constant, -6: $5(-6) = -30$. 3. Look for a factorization of -30 in which the sum of the factors is the coefficient of the middle term, 7. The numbers we need are 10 and -3. 4. Split the middle term, writing it as a sum or difference using the factors found in step (3). $7x = 10x - 3x$ 5. Factor by grouping. $5x^2 + 7x - 6$ $= 5x^2 + 10x - 3x - 6$ $= 5x(x + 2) - 3(x + 2)$ $= (5x - 3)(x + 2)$	20. Factor $8x^2 - 2x - 1$ by grouping. A. One factor is $(x - 1)$. B. One factor is $(2x - 1)$. C. One factor is $(4x - 1)$. D. One factor is $(8x - 1)$.

Objective [4.4b] Factor trinomials of the type $ax^2 + bx + c, a \neq 1$, by the FOIL method.

Brief Procedure	Example	Practice Exercise
1. Factor out the largest common factor. 2. Find the **F**irst terms whose product is ax^2. 3. Find two **L**ast terms whose product is c. 4. Repeat steps (2) and (3) until a combination is found for which the sum of the **O**uter and **I**nner products is bx.	Factor $2y^3 + 5y^2 - 3y$. 1. Factor out the largest common factor, y: $y(2y^2 + 5y - 3)$. Now we factor $2y^2 + 5y - 3$. 2. Because $2y^2$ factors as $2y \cdot y$, we have this possibility for a factorization: $(2y+ \quad)(y+ \quad)$. 3. There are two pairs of factors of -3 and each can be written in two ways: $3, -1 \quad -3, 1$ and $-1, 3 \quad 1, -3$. 4. From steps (2) and (3) we see that there are 4 possibilities for factorizations. We look for Outer and Inner products for which the sum is the middle term, $5y$. We try some possibilities. $(2y + 3)(y - 1) = 2y^2 + y - 3$ $(2y - 1)(y + 3) = 2y^2 + 5y - 3$ The factorization of $2y^2 + 5y - 3$ is $(2y - 1)(y + 3)$. We must include the common factor to get a factorization of the original trinomial. $2y^3 + 5y^2 - 3y = y(2y - 1)(y + 3)$	21. Factor $6z^2 + 14z + 4$. A. $(3z + 1)(z + 2)$ B. $2(3z + 1)(z + 2)$ C. $(6z + 1)(z + 6)$ D. $(3z + 2)(2z + 3)$

Objective [4.5a] Factor trinomial squares.

Brief Procedure	Example	Practice Exercise
Use the following equations: $A^2 + 2AB + B^2 = (A + B)^2$, $A^2 - 2AB + B^2 = (A - B)^2$	Factor $4x^2 - 12xy + 9y^2$. $4x^2 - 12xy + 9y^2$ $= (2x)^2 - 2 \cdot 2x \cdot 3y + (3y)^2$ $= (2x - 3y)^2$	22. Factor $16x^2 + 8x + 1$. A. $(2x + 1)^2$ B. $(4x + 1)^2$ C. $(8x + 1)^2$ D. $(8x - 1)^2$

Objective [4.5b] Factor differences of squares.

Brief Procedure	Example	Practice Exercise
Use the equation $A^2 - B^2 = (A + B)(A - B)$.	Factor $t^5 - t$. $t^5 - t = t(t^4 - 1)$ $= t(t^2 + 1)(t^2 - 1)$ $= t(t^2 + 1)(t + 1)(t - 1)$	23. Factor $4a^2 - 9b^2$ completely. A.$(2a - 3b)^2$ B. $(2a + 3b)(2a - 3b)$ C. $(2a + b)(2a - 9b)$ D. $(3b + 2a)(3b - 2a)$

Objective [4.5c] Factor certain polynomials with four terms by grouping and possibly using the factoring of a trinomial square or the difference of squares.		
Brief Procedure	Example	Practice Exercise
Sometimes when factoring by grouping, we get a factor that can be factored further as a difference of squares. Furthermore, we can sometimes factor a polynomial with four terms by grouping the terms as a trinomial square minus a squared term and then factoring as a difference of squares.	Factor completely. a) $x^3 + 3x^2 - x - 3$ b) $a^2 - 8a + 16 - b^2$ *a)* $x^3 + 3x^2 - x - 3$ $= x^2(x + 3) - (x + 3)$ $= (x^2 - 1)(x + 3)$ $= (x + 1)(x - 1)(x + 3)$ *b)* $a^2 - 8a + 16 - b^2$ $= (a - 4)^2 - b^2$ $= (a - 4 + b)(a - 4 - b)$	24. Factor completely: $x^2 - 2x + 1 - 9y^2$ A. $(x - 1 + 3y)(x - 1 - 3y)$ B. $(x + 1 + 3y)(x + 1 - 3y)$ C. $(x + y)(x - 2y)$ D. $(x - y)(x - 9y)$

Objective [4.5d] Factor sums and differences of cubes.		
Brief Procedure	Example	Practice Exercise
Use the following equations. $A^3 + B^3 =$ $(A + B)(A^2 - AB + B^2)$ $A^3 - B^3 =$ $(A - B)(A^2 + AB + B^2)$	Factor $y^3 - 8z^3$. $y^3 - 8z^3$ $= y^3 - (2z)^3$ $= (y - 2z)(y^2 + 2yz + 4z^2)$	25. Factor: $27m^3 + n^3$. A. $(3m + n)^3$ B. $(3m + n)(9m^2 + 3mn + n^2)$ C. $(3m + n)(9m^2 - 3mn + n^2)$ D. $(3m + n)(9m^2 - 6mn + n^2)$

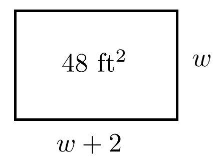
Objective [4.6a] Factor polynomials completely using any of the methods considered in this chapter.		
Brief Procedure	**Example**	**Practice Exercise**
a) Always look for a common factor. If there is one, factor out the largest common factor. b) Then look at the number of terms. *Two terms:* Determine whether you have a difference of squares or a sum or difference of cubes. Do not try to factor a sum of squares: $A^2 + B^2$. *Three terms:* Determine whether the trinomial is a square. If it is, factor accordingly. If not, try trial and error, using FOIL or grouping. *Four terms:* Try factoring by grouping. c) *Always factor completely.* If a factor with more than one term can still be factored, you should factor it. When no factor can be factored further, you have finished.	Factor $2y^3 - 12y^2 + 18y$ completely. a) We look for a common factor. $2y^3 - 12y^2 + 18y = 2y(y^2 - 6y + 9)$ b) The factor $y^2 - 6y + 9$ has three terms and is a trinomial square. We factor it. $2y(y^2 - 6y + 9)$ $= 2y(y^2 - 2 \cdot y \cdot 3 + 3^2)$ $= 2y(y - 3)^2$	26. Factor $15x^2 + 5xy - 20y^2$ completely. A. One factor is $(3x + 4y)$. B. One factor is $(3x - 4y)$. C. One factor is $(5x - 5y)$. D. One factor is $(15x + 20y)$.

Objective [4.7a] Solve quadratic and other polynomial equations by first factoring and then using the principle of zero products.		
Brief Procedure	**Example**	**Practice Exercise**
1. Obtain a 0 on one side of the equation. 2. Factor the other side. 3. Set each factor equal to 0. 4. Solve the resulting equations.	Solve: $x^2 + 2x = 24$. $x^2 + 2x = 24$ $x^2 + 2x - 24 = 0$ $(x + 6)(x - 4) = 0$ $x + 6 = 0$ *or* $x - 4 = 0$ $x = -6$ *or* $x = 4$ The solutions are -6 and 4.	27. Solve: $16x^2 = 49$. A. $\frac{7}{4}, -\frac{7}{4}$ B. $\frac{4}{7}, -\frac{4}{7}$ C. $7, -7$ D. $4, -4$

Objective [4.7b] Solve applied problems involving quadratic and other polynomial equations that can be solved by factoring.

<table>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Use the five-step problem solving process.</td>
<td>The length of a rectangular rug is 2 ft greater than the width. The area of the rug is 48 ft^2. Find the length and width.

1. *Familiarize.* We make a drawing. Let $w =$ the width of the rug. Then the length is $w + 2$.



Recall that the area of a rectangle is length $\times$ width.

2. *Translate.* We reword the problem.
Length $\times$ width is 48 ft^2.
$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
$(w + 2) \times \quad w \quad = \quad 48$

3. *Solve.* We solve the equation.
$(w + 2) \times w = 48$
$w^2 + 2w = 48$
$w^2 + 2w - 48 = 0$
$(w + 8)(w - 6) = 0$
$w + 8 = 0 \quad \textit{or} \quad w - 6 = 0$
$w = -8 \quad \textit{or} \quad w = 6$

4. *Check.* The width of a rectangle cannot be negative, so -8 cannot be a solution. Suppose the width is 6 ft. Then the length is $6 + 2$, or 8 ft and the area is $6 \cdot 8$, or 48 ft^2. These numbers check in the original problem.
5. *State.* The length is 8 ft and the width is 6 ft.</td>
<td>28. The height of a triangle is 4 cm greater than the base. The area is 30 cm^2. Find the height and the base.

A. The height is 6 cm.

B. The height is 10 cm.

C. The base is 10 cm.

D. The base is 14 cm.</td>
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