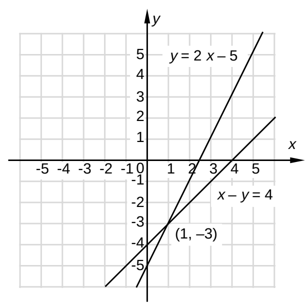
Intermediate Algebra
Chapter 3 Review

Objective [3.1a] Solve a system of two linear equations or two functions by graphing and determine whether a system is consistent or inconsistent or whether it is dependent or independent.

Brief Procedure	Example	Practice Exercise
Graph both equations and find the coordinates of the point(s) of intersection, if any exist. If the graphs are parallel lines, there is no point of intersection and, hence, no solution. If the equations have the same graph, there are infinitely many points of intersection and, thus, infinitely many solutions. Otherwise, there is exactly one point of intersection and, hence, exactly one solution. A system of equations that has a solution is consistent. A system that has no solution is inconsistent. A system of two equations that has infinitely many solutions is dependent. A system that has one solution or no solutions is independent.	Solve the system of equations graphically. Then classify the system as consistent or inconsistent and as dependent or independent. $x - y = 4,$ $y = 2x - 5$ We graph the equations.  The point of intersection appears to be $(1, -3)$. This checks in both equations, so it is the solution. The system has one solution, so it is consistent and independent.	1. Solve the system of equations graphically. Then classify the system as consistent or inconsistent and as dependent or independent. $3x - 2y = 6,$ $x - y = 1$ A. $(4, 3)$; consistent; independent B. $(2, 0)$; consistent; independent C. No solution; inconsistent; independent D. Infinitely many solutions; consistent; dependent

Objective [3.2a] Solve a system of equations in two variables by the substitution method.		
Brief Procedure	**Example**	**Practice Exercise**
If one equation has a variable alone on one side, substitute for that variable in the other equation, obtaining an equation in one variable. Solve that equation; then substitute in either original equation to find the other variable. If neither equation has a variable alone on one side, solve one equation for one of the variables. Then proceed as described above.	Solve the system $x - 2y = 1, \quad (1)$ $2x - 3y = 3. \quad (2)$ We solve Equation (1) for x, since the coefficient of x is 1 in that equation. $x - 2y = 1$ $x = 2y + 1 \quad (3)$ Now substitute for x in Equation (2) and solve for y. $2x - 3y = 3$ $2(2y + 1) - 3y = 3$ $4y + 2 - 3y = 3$ $y + 2 = 3$ $y = 1$ Now substitute 1 for y in Equation (1), (2), or (3) and find x. We choose Equation (3) since it is already solved for x. $x = 2y + 1 = 2 \cdot 1 + 1 = 2 + 1 = 3$ The ordered pair $(3, 1)$ checks in both equations, so it is the solution of the system of equations.	2. Solve the system $x + y = 3,$ $5x + 2y = 3.$ A. The x-value is -1. B. The x-value is 4. C. The x-value is -3. D. The x-value is 1.

<table>
<tr><td colspan="3">Objective [3.2b] Solve a system of two equations in two variables by the elimination method.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>If the equations have a pair of terms that are opposites, add the corresponding sides of the equations to eliminate a variable. Solve for that variable. Then substitute in either of the original equations to find the other variable.
If there is not a pair of terms that are opposites, multiply one or both equations by appropriate constants to find equivalent equations with a pair of terms that are opposites. Then proceed as described above.</td>
<td>Solve the system
$2a - 3b = 7, \quad (1)$
$3a - 2b = 8. \quad (2)$
We could eliminate either a or b. Here we decide to eliminate the a-terms. Multiply Equation (1) by 3 and Equation (2) by -2. Then add and solve for b.
$$\begin{array}{rrl} 6a - & 9b & = 21 \\ -6a + & 4b & = -16 \\ \hline & -5b & = 5 \\ & b & = -1 \end{array}$$
Next substitute -1 for b in either of the original equations.
$$\begin{aligned} 2a - 3b &= 7 \qquad (1) \\ 2a - 3(-1) &= 7 \\ 2a + 3 &= 7 \\ 2a &= 4 \\ a &= 2 \end{aligned}$$
The ordered pair $(2, -1)$ checks in both equations, so it is a solution of the system of equations.</td>
<td>3. Solve the system
$3x + 2y = 5,$
$x - y = 5.$
A. The y-value is -4.
B. The y-value is -2.
C. The y-value is 2.
D. The y-value is 3.</td>
</tr>
</table>

Objective [3.2c] Solve applied problems by solving systems of two equations using substitution or elimination.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	Two angles are supplementary. (Supplementary angles are angles whose sum is 180°.) The difference between twice one angle and the other angle is 30°. Find the angles. 1. *Familiarize.* We let x and y represent the angles. 2. *Translate.* We know that the sum of the angles is 180°. This gives us one equation. $\begin{array}{ccc} \underbrace{\text{The sum of the angles}} & \text{is} & 180^\circ. \\ \downarrow & \downarrow & \downarrow \\ x+y & = & 180 \end{array}$ We use the additional information given to translate to a second equation. $\begin{array}{ccccc} \underbrace{\text{Twice one angle}} & \text{less} & \underbrace{\text{the other}} & \text{is} & 30^\circ. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2x & - & y & = & 30 \end{array}$ We now have a system of equations: $x + y = 180$ $2x - y = 30.$ 3. *Solve.* We will use the elimination method. First we add the equations to eliminate the y-terms. $\begin{array}{rl} x + y &= 180 \\ 2x - y &= 30 \\ \hline 3x \quad &= 210 \\ x &= 70 \end{array}$ Now we substitute in one of the original equations to find y. We use the first equation. $\begin{aligned} x + y &= 180 \\ 70 + y &= 180 \\ y &= 110 \end{aligned}$ 4. *Check.* The sum of 70° and 110° is 180°. Also, $2 \cdot 70^\circ - 110^\circ = 140^\circ - 110^\circ = 30^\circ$, so the answer checks. 5. *State.* The angles are 70° and 110°.	4. The sum of two numbers is −3. The sum of twice one number and the other is 4. Find the numbers. A. One number is −10. B. One number is −7. C. One number is −4. D. One number is 4.

Objective [3.3a] Solve applied problems involving total value and mixture using systems of two equations.

Brief Procedure	Example
Use the five-step problem solving process.	Solution A is 40% acid and solution B is 55% acid. How much of each should be used in order to make 100 L of a solution that is 46% acid?

1. *Familiarize.* Let x and y represent the number of liters of 40% and 55% solution to be used, respectively. We organize the given information in a table.

Type of solution	A	B	Mixture
Amount of solution	x	y	100 L
Percent of acid	40%	55%	46%
Amount of acid in solution	$40\%x$	$55\%y$	$46\% \times 100$, or 46 L

2. *Translate.* The first row of the table gives us one equation.

$$x + y = 100$$

The last row gives us a second equation.

$$40\%x + 55\%y = 46, \text{ or}$$
$$0.4x + 0.55y = 46$$

After multiplying by 100 on both sides of the second equation to clear decimals, we have the following system of equations.

$$\begin{aligned} x + \quad y &= 100, \qquad (1) \\ 40x + 55y &= 4600 \qquad (2) \end{aligned}$$

3. *Solve.* We use the elimination method. First multiply Equation (1) by -40 and then add to eliminate the x-terms.

$$\begin{aligned} -40x - 40y &= -4000 \\ 40x + 55y &= 4600 \\ \hline 15y &= 600 \\ y &= 40 \end{aligned}$$

Now substitute in Equation (1) and solve for x.

$$\begin{aligned} x + y &= 100 \\ x + 40 &= 100 \\ x &= 60 \end{aligned}$$

4. *Check.* The sum of 60 and 40 is 100. Also, 40% of 60 L is 24 L and 55% of 40 L is 22 L. These add up to 46 L, so the answer checks.

5. *State.* 60 L of solution A and 40 L of solution B should be used.

Practice Exercise

5. There were 220 tickets sold for a school play. The price for students was \$3 and it was \$7 for non-students. A total of \$1080 was collected. How many of each type of ticket were sold?

A. Student: 75, non-student: 145
B. Student: 90, non-student: 130
C. Student: 95, non-student: 125
D. Student: 115, non-student: 105

Objective [3.3b] Solve applied problems involving motion using systems of two equations.

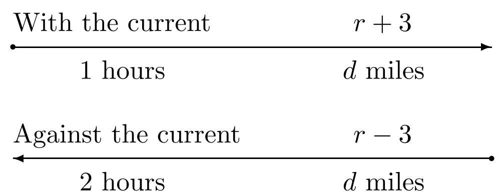
Brief Procedure

Use the five-step problem solving process. It is often convenient to translate to a system of equations.

Example

A canoeist paddled for 1 hr with a 3 mph current. The return trip against the current took 2 hr. Find the speed of the canoe in still water.

1. *Familiarize.* We first make a drawing. Let $d =$ the distance traveled in one direction and let $r =$ the speed of the canoe in still water.When the canoe travels with the current, its speed is $r + 3$ and traveling against the current the speed is $r - 3$.



We can also organize the given information in a table.

$d \quad = \quad r \quad \cdot \quad t$

	Distance	Speed	Time
With current	d	$r + 3$	1
Against current	d	$r - 3$	2

2. *Translate.* Using $d = rt$, we get an equation from each row of the table.

$d = (r + 3)1, \qquad (1)$
$d = (r - 3)2 \qquad (2)$

3. *Solve.* We use the substitution method, substituting $(r - 3)2$ for d in Equation (1).

$$
\begin{aligned}
(r-3)2 &= (r+3)1 \\
2r - 6 &= r + 3 \\
r - 6 &= 3 \\
r &= 9
\end{aligned}
$$

4. *Check.* When $r = 9$, then $r + 3 = 12$ and $12 \cdot 1 = 12$, the distance traveled with the current. When $r = 9$, then $r - 3 = 6$ and $6 \cdot 2 = 12$, the distance traveled against the current. Since the distances are the same, the answer checks.

5. *State.* The speed of the canoe in still water is 9 mph.

Practice Exercise

6. A train leaves a station and travels west at 80 mph. One hour later a second train leaves the same station and travels west on a parallel track at 100 mph. When will it overtake the first train?

A. 4 hr after the first train leaves
B. 5 hr after the first train leaves
C. 6 hr after the first train leaves
D. 8 hr after the first train leaves

Objective [3.4a] Solve systems of three equations in three variables.		
Brief Procedure	Example	Practice Exercise
1. Write all equations in the standard form $Ax + By + Cz = D$. 2. Clear any decimals or fractions. 3. Choose a variable to eliminate. Then use *any* two of the three equations to get an equation in two variables. 4. Next, use a different pair of equations and get another equation in *the same two variables*. That is, eliminate the same variable that you did in step (3). 5. Solve the resulting system (pair) of equations. That will give two of the numbers. 6. Then use any of the original three equations to find the third number.	Solve: $x - y - z = -2, \quad (1)$ $2x + 3y + z = 3, \quad (2)$ $5x - 2y - 2z = -1 \quad (3)$ The equations are in standard form and do not contain decimals or fractions. We will choose to eliminate z since the z-terms in equations (1) and (2) are opposites. First we add these two equations. $x - y - z = -2$ $2x + 3y + z = 3$ $3x + 2y = 1 \quad (4)$ Next we multiply equation (2) by 2 and add it to equation (3) to eliminate z from another pair of equations. $4x + 6y + 2z = 6$ $5x - 2y - 2z = -1$ $9x + 4y = 5 \quad (5)$ Now we solve the system consisting of equations (4) and (5). We multiply equation (4) by -2 and add. $-6x - 4y = -2$ $9x + 4y = 5$ $3x = 3$ $x = 1$ Now use either equation (4) or (5) to find y. $3x + 2y = 1 \quad (4)$ $3 \cdot 1 + 2y = 1$ $3 + 2y = 1$ $2y = -2$ $y = -1$ Finally, use one of the original equations to find z. $2x + 3y + z = 3 \quad (2)$ $2 \cdot 1 + 3(-1) + z = 3$ $-1 + z = 3$ $z = 4$ The ordered triple $(1, -1, 4)$ checks in all three equations, so it is the solution of the system of equations.	7. Solve: $x - y + z = 9,$ $2x + y + 2z = 3,$ $4x + 2y - 3z = -1$ A. The y-value is -5. B. The y-value is -3. C. The y-value is 1. D. The y-value is 3.

<table>
<tr><td colspan="3">Objective [3.5a] Solve applied problems using systems of three equations.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Use the five-step problem solving process.</td>
<td>In triangle ABC, the measure of angle B is three times that of angle A. The measure of angle C is 40° more than that of angle B. Find the measure of each angle.

1. Familiarize. Let $x =$ the measure of angle A, $y =$ the measure of angle B, and $z =$ the measure of angle C.

2. Translate. The sum of the measures of the angles of a triangle is 180°, so this gives us one equation:
$x + y + z = 180.$
We can also translate two statements in the problem directly.

$\underbrace{\text{The measure of angle } B}_{y} \; \underset{=}{\text{is}} \; \underset{3\cdot}{\text{three times}} \; \underbrace{\text{that of angle } A.}_{x}$

$\underbrace{\text{The measure of angle } C}_{z} \; \underset{=}{\text{is}} \; \underset{40}{40^\circ} \; \underset{+}{\text{more than}} \; \underbrace{\text{that of angle } B.}_{y}$

3. Solve. Solving the system of equations, we get (20, 60, 100).

4. Check. The sum of the measures is 20° +60° +100°, or 180°. The measure of angle B, 60°, is 3 times the measure of angle A, 20°, and the measure of angle C, 100°, is 40° more than the measure of angle B. The answer checks.

5. State. The measure of angle A is 20°, the measure of angle B is 60°, and the measure of angle C is 100°.</td>
<td>8. Maggie, Juliet, and Erik can process 164 telephone orders per day. Maggie and Juliet together can process 109 orders, while Juliet and Erik together can process 116 orders. How many orders can each person process alone?

A. Maggie can process 48 orders alone.

B. Juliet can process 62 orders alone.

C. Erik can process 56 orders alone.

D. Erik can process 60 orders alone.</td>
</tr>
</table>

<table>
<tr><td colspan="3">Objective [3.6a] Determine whether an ordered pair of numbers is a solution of an inequality in two variables.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr><td>Following alphabetical order, substitute the coordinates of the ordered pair in the inequality and determine whether a true inequality results.</td><td>Determine whether $(4, -1)$ is a solution of $x + 3y \geq 5$.

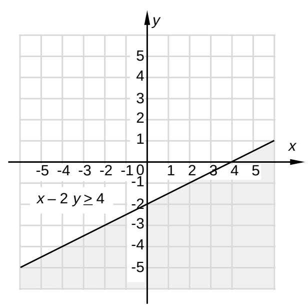
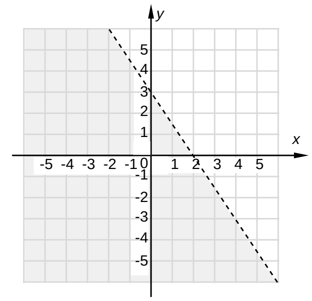
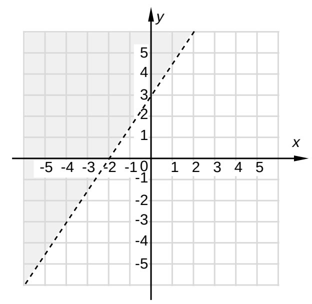
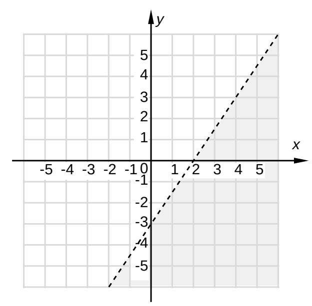
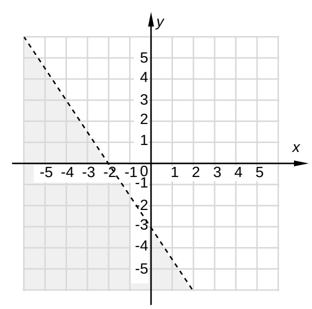
Use alphabetical order to replace x with 4 and y with -1.

$$\begin{array}{r|l} x + 3y & \geq 5 \\ \hline 4 + 3(-1) & ?\ 5 \\ 4 - 3 & \\ 1 & \qquad \text{FALSE} \end{array}$$
Since $1 \geq 5$ is false, $(4, -1)$ is not a solution.</td><td>9. Determine whether $(-2, 5)$ is a solution of $3x + y \leq -1$.

A. Yes

B. No</td></tr>
</table>

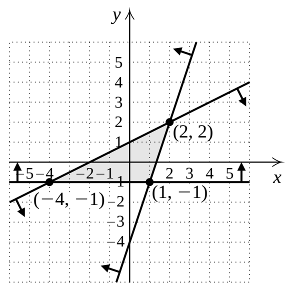
Objective [3.6b] Graph linear inequalities in two variables.

Brief Procedure	Example	Practice Exercise
1. Replace the inequality symbol with an equals sign and graph this related equation. 2. If the inequality symbol is $<$ or $>$, draw the line dashed. If the inequality symbol is $\leq$ or $\geq$, draw the line solid. 3. The graph consists of a half-plane, either above or below or left or right of the line, and, if the line is solid, the line as well. To determine which half-plane to shade, choose a point not on the line as a test point. Substitute to find whether that point is a solution of the inequality. If it is, shade the half-plane containing that point. If it is not, shade the half-plane on the opposite side of the line.	Graph: $x - 2y \geq 4$. First graph the line $x - 2y = 4$. The intercepts are $(0, -2)$ and (4,0). We draw the line solid since the inequality symbol is $\geq$. Next, choose a test point not on the line and determine if it is a solution of the inequality. We choose (0,0), since it is usually an easy point to use. $\begin{array}{c\|c} \multicolumn{2}{c}{x - 2y \geq 4} \\ \hline 0 - 2 \cdot 0 & ?\ 4 \\ 0 & \quad \text{FALSE} \end{array}$ Since (0,0) is not a solution, we shade the half-plane that does not contain (0,0). 	10. Graph $3x + 2y < -6$. A.  B.  C.  D. 

<table>
<tr><td colspan="3">Objective [3.6c] Graph systems of linear inequalities and find coordinates of any vertices.</td></tr>
<tr><td>Brief Procedure</td><td>Example</td><td>Practice Exercise</td></tr>
<tr>
<td>Graph each inequality and determine where the graphs overlap, or intersect. The solutions of the system of inequalities are the ordered pairs in this region.
To find the vertices, solve systems of equations composed of the appropriate related equations.</td>
<td>Graph the system of inequalities and find the coordinates of any vertices formed.

$$\begin{aligned} x-2y &\geq -2, \quad (1)\\ 3x-y &\leq 4, \quad (2)\\ y &\geq -1, \quad (3) \end{aligned}$$

We graph the lines using solid lines. Indicate the region for each inequality by arrows at the ends of the line. Shade the region of overlap.



To find the vertices, we solve three different systems of equations. From inequalities (1) and (2) we have

$$\begin{aligned} x-2y &= -2,\\ 3x-y &= 4 \end{aligned}$$

Solving, we obtain the vertex (2, 2).
From inequalities (1) and (3) we have

$$\begin{aligned} x-2y &= -2,\\ y &= -1 \end{aligned}$$

Solving, we obtain the vertex $(-4,-1)$.
From inequalities (2) and (3) we have

$$\begin{aligned} 3x-y &= 4,\\ y &= -1 \end{aligned}$$

Solving, we obtain the vertex $(1,-1)$.</td>
<td>11. Graph the system of inequalities and find the coordinates of any vertices found.

$$\begin{aligned} x-2y &\leq 4,\\ x+y &\leq 4,\\ x-1 &\geq 0 \end{aligned}$$

A. One vertex is $(0,-4)$.
B. One vertex is $(1,0)$.
C. One vertex is $(1,3)$.
D. One vertex is $(8,-4)$.</td>
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</table>

Objective [3.7a] Given total-cost and total-revenue functions, find the total-profit function and the break-even point.		
Brief Procedure	Example	Practice Exercise
Given a total-cost function $C(x)$ and a total-revenue function $R(x)$, the total-profit function $P(x)$ is given by $P(x) = R(x) - C(x)$. To find the break-even point, solve the system of equations composed of $C(x)$ and $R(x)$.	For the total-cost function $C(x) = 10x + 250,000$ and the total-revenue function $R(x) = 60x$, find (a) the total-profit function and (b) the break-even point. a) Total profit is given by $P(x) = R(x) - C(x)$ $= 60x - (10x + 250,000)$ $= 60x - 10x - 250,000$ $= 50x - 250,000.$ b) To find the break-even point we solve the system of equations $C(x) = 10x + 250,000,$ $R(x) = 60x.$ Since both cost and revenue are in dollars and they are equal at the break-even point, the system can be rewritten as $d = 10x + 250,000,$ $d = 60x.$ Then we solve using the substitution method. We substitute $60x$ for d in the first equation and solve for x. $60x = 10x + 250,000$ $50x = 250,000$ $x = 5000$ Now substitute in either equation to find the second coordinate of the break-even point. We will use the second equation. $d = 60x$ $d = 60 \cdot 5000$ $d = 300,000$ The break-even point is (5000, \$300,000).	12. For the total-cost function $C(x) = 15x + 400,000$ and the total-revenue function $R(x) = 55x$, find the total-profit function and the break-even point. A. $P(x) = 400,000 - 40x$ B. $P(x) = 40x + 400,000$ C. The equilibrium point is (10,000, \$550,000). D. The equilibrium point is (4000, \$220,000).

Objective [3.7b] Given supply and demand functions, find the equilibrium point.		
Brief Procedure	Example	Practice Exercise
Solve the system of equations composed of the supply and demand functions.	Find the equilibrium point for the demand and supply functions $D(p) = 5000 - 30p,$ $S(p) = 2000 + 10p.$ Since both demand and supply are *quantities* and they are equal at the equilibrium point, we rewrite the system as $q = 5000 - 30p;$ $q = 2000 + 10p.$ Then substitute $2000 + 10p$ for q in the first equation and solve for p. $2000 + 10p = 5000 - 30p$ $2000 + 40p = 5000$ $40p = 3000$ $p = 75$ The equilibrium price is \$75 per unit. To find the equilibrium quantity, we substitute 75 into either $D(p)$ or $S(p)$. $S(p) = 2000 + 10(75)$ $= 2000 + 750$ $= 2750$ The equilibrium quantity is 2750 units and the equilibrium point is (\$75, 2750).	13. Find the equilibrium point for the demand and supply functions $D(p) = 900 - 17p,$ $S(p) = 150 + 8p.$ A. (\$30, 390) B. (\$45, 135) C. (\$60, 630) D. (\$125, 1150)