Intermediate Algebra
Chapter 1 Review

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<tr><td colspan="3">Objective [1.1a] Determine whether a given number is a solution of a given equation.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr><td>Substitute the given number in the equation and determine if a true equation results.</td><td>Determine whether -3 is a solution of $x+4=1$.
$$\begin{array}{c|c} x+4 & 1 \\ \hline -3+4 \ ? & 1 \\ 1 & \end{array} \quad \text{TRUE}$$
Since the left-hand and right-hand sides are the same, we have a true equation so -3 is a solution.</td><td>1. Determine whether 5 is a solution of $9x=42$.
A. Yes
B. No</td></tr>
<tr><td colspan="3">Objective [1.1b] Solve equations using the addition principle.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr><td>For any real numbers a, b, and c,
$a=b$ is equivalent to
$a+c=b+c$.
Add the same number on both sides of the equation to get the variable alone. Since $a+(-c)=b+(-c)$ is equivalent to $a-c=b-c$, we can also subtract the same number on both sides of the equation.</td><td>Solve: $x+4=9$.

We subtract 4 on both sides of the equation to get x alone.
$$\begin{aligned} x+4&=9 \\ x+4-4&=9-4 \\ x+0&=5 \\ x&=5 \end{aligned}$$
The solution is 5.</td><td>2. Solve: $y-3=-1$.
A. -4
B. -2
C. 2
D. 4</td></tr>
<tr><td colspan="3">Objective [1.1c] Solve equations using the multiplication principle.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr><td>For any real numbers a, b, and c, $c \neq 0$,
$a=b$ is equivalent to
$a\cdot c=b\cdot c$.
Multiply by the same number on both sides of the equation to get the variable alone. For $c\neq 0$, $a\cdot\frac{1}{c}=b\cdot\frac{1}{c}$ is equivalent to $\frac{a}{c}=\frac{b}{c}$, so we can also divide by the same number on both sides of the equation.</td><td>Solve: $54=-9y$.

We divide by -9 on both sides of the equation to get y alone.
$$\begin{aligned} 54&=-9y \\ \frac{54}{-9}&=\frac{-9y}{-9} \\ -6&=1\cdot y \\ -6&=y \end{aligned}$$
The solution is -6.</td><td>3. Solve: $6x=-42$.
A. 7
B. -7
C. -36
D. -48</td></tr>
</table>

Objective [1.1d] Solve equations using the addition and the multiplication principles together, removing parentheses where appropriate.		
Brief Procedure	Example	Practice Exercise
First use the addition principle to isolate the term that contains the variable. Then use the multiplication principle to get the variable by itself. If an equation contains parentheses, first use the distributive laws to remove them. Then collect like terms, if necessary, and use the addition and multiplication principles to complete the solution of the equation.	Solve: $8b - 2(3b+1) = 10$. $8b - 2(3b+1) = 10$ $8b - 6b - 2 = 10$ $2b - 2 = 10$ $2b - 2 + 2 = 10 + 2$ $2b = 12$ $\frac{2b}{2} = \frac{12}{2}$ $b = 6$ The solution is 6.	4. Solve: $3(n-4) = 2(n+1)$. A. -5 B. -2 C. 5 D. 14

Objective [1.2a] Evaluate formulas and solve a formula for a specified letter.		
Brief Procedure	Example	Practice Exercises
To evaluate a formula for a given value of the variable, substitute the value for the variable and carry out the resulting calculations.	The formula $d = 65t$ gives the distance d that is traveled in t hours at a speed of 65 mph. Suppose you travel at 65 mph for 4 hours. How far have you traveled? We substitute 4 for t and carry out the calculation. $d = 65 \cdot 4 = 260$ You have traveled 260 miles.	5. Using the formula $d = 65t$, find the distance traveled at 65 mph for 3 hours. A. 185 miles B. 195 miles C. 205 miles D. 225 miles
To solve a formula for a given letter, identify the letter and: 1. Multiply on both sides to clear the fractions or decimals, if necessary. 2. If parentheses occur, multiply to remove them using the distributive law. 3. Collect like terms on each side, if necessary. This may require factoring if a variable is in more than one term. 4. Using the addition principle, get all terms with the letter to be solved for on one side of the equation and all other terms on the other side. 5. Collect like terms again, if necessary. 6. Solve for the letter in question using the multiplication principle.	Solve for b: $A = \frac{a+b}{2}$. $A = \frac{a+b}{2}$ $2 \cdot A = 2\left(\frac{a+b}{2}\right)$ $2A = a + b$ $2A - a = b$	6. Solve for h: $A = \frac{1}{2}bh$. A. $h = \frac{A}{2b}$ B. $h = \frac{b}{2A}$ C. $h = \frac{2b}{A}$ D. $h = \frac{2A}{b}$

<table>
<tr><td colspan="3">Objective [1.3a] Solve applied problems by translating to equations.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Use the five-step problem solving process.
1. Familiarize yourself with the problem situation.
2. Translate the problem to an equation.
3. Solve the equation.
4. Check the answer in the original problem.
5. State the answer to the problem clearly.</td>
<td>A 12-ft pipe is cut into three pieces. The second piece is three times as long as the first. The third piece is twice as long as the first. How long is each piece?

1. Familiarize. Let $x =$ the length of the first piece of pipe. Then $3x =$ the length of the second piece and $2x =$ the length of the third piece.

2. Translate. We use the fact that the sum of the lengths is 12 feet.

$$\underbrace{\text{Length of first piece}}_{\downarrow \atop x}\ \ \underset{\downarrow \atop +}{\text{plus}}\ \ \underbrace{\text{length of second piece}}_{\downarrow \atop 3x}\ \ \underset{\downarrow \atop +}{\text{plus}}$$
$$\underbrace{\text{length of third piece}}_{\downarrow \atop 2x}\ \ \underset{\downarrow \atop =}{\text{is}}\ \ \underbrace{\text{total length}}_{\downarrow \atop 12}$$
3. Solve. We solve the equation.
$$\begin{aligned} x + 3x + 2x &= 12 \\ 6x &= 12 \\ \frac{6x}{6} &= \frac{12}{6} \\ x &= 2 \end{aligned}$$
If $x = 2$, then $3x = 3 \cdot 2$, or 6, and $2x = 2 \cdot 2$, or 4.

4. Check. The second piece, 6 ft, is three times as long as the first, 2 ft, and the third piece, 4 ft, is twice as long as the first. Also, the lengths total 2 ft + 6 ft + 4 ft, or 12 ft. The answer checks.
5. State. The first piece of pipe is 2 ft long, the second piece is 6 ft, and the third piece is 4 ft.</td>
<td>7. The perimeter of a rectangular rug is 40 ft. The width is 4 ft less than the length. Find the dimensions of the rug.
A. The width is 8 ft.
B. The width is 10 ft.
C. The width is 12 ft.
D. The width is 16 ft.</td>
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Objective [1.3b] Solve basic motion problems.		
Brief Procedure	**Example**	**Practice Exercise**
Use the motion formula, $d = rt$, and the five-step problem solving process.	An airplane that travels 425 mph in still air encounters a 25-mph headwind. How long will it take to travel 600 mi into the wind? 1. *Familiarize.* We let $t =$ the number of hours it will take the plane to travel 600 mi into the wind. The plane's speed flying into the wind is $425 - 25$, or 400 mph. 2. *Translate.* We use the motion formula $d = rt$ and substitute 600 for d and 400 for r. $d = rt$ $600 = 400 \cdot t$ 3. *Solve.* We solve the equation. $600 = 400t$ $\frac{600}{400} = \frac{400t}{400}$ $1.5 = t$ 4. *Check.* At a speed of 400 mph, in a time of 1.5 hr the plane would travel 400(1.5), or 600 mi. The answer checks. 5. *State.* It will take 1.5 hr to travel 600 mi into the headwind.	8. A boat travels at a rate of 16 km/h in still water. If a river's current moves at a rate of 4 km/h, how long will it take the boat to travel 60 km downstream? A. 3 hr B. 3.75 hr C. 5 hr D. 15 hr

Objective [1.4a] Determine whether a given number is a solution of an inequality.		
Brief Procedure	**Example**	**Practice Exercise**
Substitute the given number for the variable and determine if a true inequality results.	Determine whether each number is a solution of $y \leq -4$. a) 2 b) -4 a) Since $2 \leq -4$ is false, 2 is not a solution. b) Since $-4 \leq -4$ is true, -4 is a solution.	9. Determine whether -3 is a solution of $x \geq -5$. A. Yes B. No

Objective [1.4b] Write interval notation for the solution set or graph of an inequality.

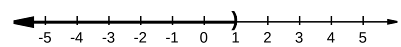
Brief Procedure

To indicate that a solution set contains all the points in the interval from a to b, write (a, b). The parentheses indicate that neither a nor b is in the solution set. Use a bracket to signify that an endpoint is included. For example, $[a, b) = \{x | a \le x < b\}$. Use the symbols $-\infty$ and ∞ to indicate that an interval extends without bound to the left or to the right, respectively.

Example

Write interval notation for the set or graph.

a) $\{x | -3 < x \le 2\}$

b) 

a) The set contains all real numbers from -3 to 2 along with the right endpoint 2, so we write $(-3, 2]$.

b) The set contains all real numbers less than 1, so we write $(-\infty, 1)$.

Practice Exercise

10. Write interval notation for the set $\{x | x \ge 2.6\}$.

A. $(-\infty, 2.6)$
B. $(-\infty, 2.6]$
C. $(2.6, \infty)$
D. $[2.6, \infty)$

Objective [1.4c] Solve an inequality using the addition and multiplication principles and then graph the inequality.

Brief Procedure

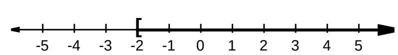
First use the addition principle to isolate the term that contains the variable. Then use the multiplication principle to get the variable by itself. Keep in mind that the inequality symbol must be reversed when the multiplication principle is used to multiply by a negative number on both sides of the inequality. Then graph the solution set on the number line.

Example

Solve: $6y + 5 \ge 3y - 1$.

$$\begin{aligned} 6y + 5 &\ge 3y - 1 \\ 6y + 5 - 3y &\ge 3y - 1 - 3y \\ 3y + 5 &\ge -1 \\ 3y + 5 - 5 &\ge -1 - 5 \\ 3y &\ge -6 \\ \frac{3y}{3} &\ge \frac{-6}{3} \\ y &\ge -2 \end{aligned}$$

The solution set is $\{y | y \ge -2\}$, or $[-2, \infty)$.
We graph the solution set.



Practice Exercise

11. Solve: $8y - 7 > 5y + 2$.

A. $\{y | y < -3\}$
B. $\{y | y > -3\}$
C. $\{y | y > \frac{9}{13}\}$
D. $\{y | y > 3\}$

Objective [1.4d] Solve applied problems by translating to inequalities.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	The perimeter of a rectangular patio is not to exceed 60 ft. The length is to be twice the width. What widths will meet these conditions? 1. *Familiarize.* Recall that the formula for the perimeter P of a rectangle is $P = 2l + 2w$, where l is the length and w is the width. Since the length is twice the width, we have $l = 2w$ and then $2l + 2w = 2 \cdot 2w + 2w = 4w + 2w = 6w$. 2. *Translate.* We reword the problem and translate. $\underbrace{\text{Perimeter}}_{\downarrow \atop 6w} \quad \underbrace{\text{is less than or equal to}}_{\downarrow \atop \leq} \quad \underbrace{60 \text{ ft.}}_{\downarrow \atop 60}$ 3. *Solve.* We solve the inequality. $6w \leq 60$ $\frac{6w}{6} \leq \frac{60}{6}$ $w \leq 10$ 4. *Check.* We can obtain a partial check by substituting a number less than or equal to 10 for w. For example, for $w = 9$, we have $l = 2 \cdot 9$, or 18 and $2l + 2w = 2 \cdot 18 + 2 \cdot 9 = 54 \leq 60$. The result is probably correct. 5. *State.* Widths less than or equal to 10 ft will meet the given conditions.	12. Kelly's scores on the first three tests in her physics class are 79, 84, and 68. Determine all scores on the fourth test that will yield an average test score of at least 80. A. Scores of 88 or higher B. Scores of 89 or higher C. Scores of 91 or higher D. Scores of 94 or higher

Objective [1.5a] Find the intersection of two sets. Solve and graph conjunctions of inequalities.

Brief Procedure	Example	Practice Exercise
The intersection of two sets A and B, denoted $A \cap B$, is the set of all numbers that are common to A and B.	Find the intersection: $\{-4, -2, 0, 2, 4\} \cap \{-2, -1, 0, 1, 2\}$. The numbers -2, 0, and 2 are common to the two sets, so the intersection is $\{-2, 0, 2\}$.	13. Find the intersection: $\{0, 3, 4, 9\} \cap \{-3, 0, 4, 8\}$. A. $\{0\}$ B. $\{3\}$ C. $\{0, 3\}$ D. $\{0, 4\}$

Objective [1.5a] (continued)

Brief Procedure

To solve a conjunction of inequalities, use the addition and multiplication principles to solve both parts of the inequality. The solution set of the conjunction is the intersection of the individual solution sets. The solution set can then be graphed on the number line.

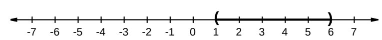
Example

Solve and graph:
a) $-1 < 2x - 3$ *and* $2x - 3 < 9$;
b) $4x + 10 > -14$ *and* $3x - 4 \leq 2$.

a) $-1 < 2x - 3$ *and* $2x - 3 < 9$
We can combine the parts of this conjunction into a single inequality.

$$-1 < 2x - 3 < 9$$
$$2 < 2x < 12$$
$$1 < x < 6$$

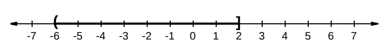
The solution set is $\{x|1 < x < 6\}$, or $(1, 6)$.
We graph the solution set.



b) The parts of this conjunction cannot be combined into a single inequality, so we solve each part separately.

$$4x + 10 > -14 \text{ and } 3x - 4 \leq 2$$
$$4x > -24 \text{ and } 3x \leq 6$$
$$x > -6 \text{ and } x \leq 2$$

Now we find the intersection of the solution sets:
$\{x|x > -6\} \cap \{x|x \leq 2\} =$
$\{x| -6 < x \leq 2\}$.
The solution set is $\{x| -6 < x \leq 2\}$, or $(-6, 2]$.
We graph the solution set.



Practice Exercise

14. Solve: $-4 \leq 5x + 6 < 11$.

A. $\left\{x \middle| \frac{2}{5} \leq x < \frac{17}{5}\right\}$, or $\left[\frac{2}{5}, \frac{17}{5}\right)$

B. $\left\{x \middle| \frac{2}{5} \leq x < 1\right\}$, or $\left[\frac{2}{5}, 1\right)$

C. $\{x| -2 \leq x < 1\}$, or $[-2, 1)$

D. $\left\{x \middle| -2 \leq x < \frac{17}{5}\right\}$, or $\left[-2, \frac{17}{5}\right)$

Objective [1.5b] Find the union of two sets. Solve and graph disjunctions of inequalities.

Brief Procedure

The union of two sets A and B, denoted $A \cup B$, is the collection of elements belonging to A and/or B.

Example

Find the union: $\{1, 3, 5\} \cup \{1, 5, 6\}$.

The numbers in either or both sets are 1, 3, 5, and 6, so the union is $\{1, 3, 5, 6\}$.

Practice Exercise

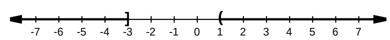
15. Find the union:
$\{-4, 2, 6\} \cap \{-3, 0, 2, 6\}$.

A. $\{2, 6\}$

B. $\{-4, 0, 2, 6\}$

C. $\{-4, -3, 2, 6\}$

D. $\{-4, -3, 0, 2, 6\}$

<table>
<tr><td colspan="3">Objective [1.5b] (continued)</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>To solve a disjunction of inequalities, use the addition and multiplication principles to solve each inequality separately, retaining the word or. The solution set of the disjunction is the union of the individual solution sets. The solution set can then be graphed on the number line.</td>
<td>Solve and graph:
$3x + 1 \le -8$ or $4x + 1 > 5$.
$$\begin{aligned} 3x + 1 \le -8 \;& \textit{or}\; 4x + 1 > 5 \\ 3x \le -9 \;& \textit{or}\; \quad 4x > 4 \\ x \le -3 \;& \textit{or}\; \quad x > 1 \end{aligned}$$
Now find the union of the solution sets:
$\{x \mid x \le -3\} \cup \{x \mid x > 1\} =$
$\{x \mid x \le -3 \textit{ or } x > 1\}$.
The solution set is
$\{x \mid x \le -3 \textit{ or } x > 1\}$, or
$(-\infty, -3] \cup (1, \infty)$.
We graph the solution set.

</td>
<td>16. Solve: $x + 4 < 2$ or $2x + 3 \ge 7$.
A. $\emptyset$
B. $(-2, 2]$
C. $(-\infty, -2) \cup [2, \infty)$
D. $(-\infty, -2) \cup [5, \infty)$</td>
</tr>
<tr><td colspan="3">Objective [1.5c] Solve applied problems involving conjunctions and disjunctions of inequalities.</td></tr>
<tr><th>Brief Procedure</th><th>Example</th><th>Practice Exercise</th></tr>
<tr>
<td>Use the five-step problem solving process.</td>
<td>The equation $P = 1 + \dfrac{d}{33}$ gives the pressure P, in atmospheres (atm) at a depth of d feet in the sea. For what depths d is the pressure at least 2 atm and at most 6 atm?
1. Familiarize. We will use the given formula, $P = 1 + \dfrac{d}{33}$.
2. Translate. We want to find the depths for which the pressure is greater than or equal to 2 and less than or equal to 6, so we have
$$2 \le 1 + \frac{d}{33} \le 6.$$
3. Solve. We solve the inequality.
$$\begin{aligned} 2 \le 1 + \frac{d}{33} &\le 6 \\ 66 \le 33 + d &\le 198 \\ 33 \le d &\le 165 \end{aligned}$$
4. Check. A partial check can be done by substituting some values of d in the formula.
5. State. The pressure is at least 2 atm and at most 6 atm for depths d, in feet, such that $\{d \mid 33 \le d \le 165\}$.</td>
<td>17. The formula $F = 1.8C + 32$ can be used to convert Celsius temperatures C to Fahrenheit temperatures F. For what Celsius temperatures are the corresponding Fahrenheit temperatures between 32° and 212°?
A. $0° < C < 100°$
B. $0° < C < 141°$
C. $35° < C < 100°$
D. $35° < C < 141°$</td>
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</table>

Objective [1.6a] Simplify expressions containing absolute-value symbols.		
Brief Procedure	Example	Practice Exercise
Use the properties of absolute value: For any real numbers a and b, a) $\lvert ab\rvert = \lvert a\rvert \cdot \lvert b\rvert$. (The absolute value of a product is the product of the absolute values.) b) $\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a\rvert}{\lvert b\rvert}$, provided that $b \neq 0$. (The absolute value of a quotient is the quotient of the absolute values.) c) $\lvert -a\rvert = \lvert a\rvert$. (The absolute value of the opposite of a number is the same as the absolute value of the number.)	Simplify: a) $\lvert -2z\rvert$; b) $\left\lvert \frac{5}{x} \right\rvert$. a) $\lvert -2z\rvert = \lvert -2\rvert \cdot \lvert z\rvert = 2\lvert z\rvert$. b) $\left\lvert \frac{5}{x} \right\rvert = \frac{\lvert 5\rvert}{\lvert x\rvert} = \frac{5}{\lvert x\rvert}$	18. Simplify: $\lvert -1.8y\rvert$. A. $-1.8y$ B. $-1.8\lvert y\rvert$ C. $1.8y$ D. $1.8\lvert y\rvert$
Objective [1.6b] Find the distance between two points on the number line.		
Brief Procedure	Example	Practice Exercise
For any real numbers a and b, the distance between them is $\lvert a-b\rvert$.	Find the distance between -5 and -13 on the number line. $\lvert -5-(-13)\rvert = \lvert -5+13\rvert = \lvert 8\rvert = 8$	19. Find the distance between -4 and 7 on the number line. A. 3 B. 4 C. 11 D. 15
Objective [1.6c] Solve equations with absolute-value expressions.		
Brief Procedure	Example	Practice Exercise
Use the absolute-value principle. For any positive number p and any algebraic expression X: a) The solutions of $\lvert X\rvert = p$ are those numbers that satisfy $X = -p$ or $X = p$. b) The equation $\lvert X\rvert = 0$ is equivalent to the equation $X = 0$. c) The equation $\lvert X\rvert = -p$ has no solution.	Solve: $\lvert x-4\rvert = 3$. $x-4=-3$ *or* $x-4=3$ $x=1$ *or* $x=7$ The solution set is $\{1,7\}$.	20. Solve: $\lvert 3x-1\rvert = 5$. A. $\{2\}$ B. $\left\{-\frac{4}{3}, 2\right\}$ C. $\left(-\frac{4}{3}, 2\right)$ D. $\{-2, 2\}$

Objective [1.6d] Solve equations with two absolute-value expressions.

Brief Procedure	Example	Practice Exercise
Consider $\lvert a\rvert = \lvert b\rvert$. This means that a and b are the same distance from 0. Thus, they are the same number or they are opposites, so we have $a = b$ or $a = -b$.	Solve: $\lvert 3x-4\rvert = \lvert x+8\rvert$. $3x-4 = x+8 \ \textit{or}\ 3x-4 = -(x+8)$ $2x-4 = 8 \ \textit{or}\ 3x-4 = -x-8$ $2x = 12 \ \textit{or}\ 4x-4 = -8$ $x = 6 \ \textit{or}\ 4x = -4$ $x = 6 \ \textit{or}\ x = -1$ The solution set if $\{-1, 6\}$.	21. Solve: $\lvert 2x+1\rvert = \lvert x+5\rvert$. A. $\{-2, 4\}$ B. $\left\{2, \frac{4}{3}\right\}$ C. $\{-2, 6\}$ D. $\left\{-2, -\frac{4}{3}\right\}$

Objective [1.6e] Solve inequalities with absolute-value expressions.

Brief Procedure	Example	Practice Exercise
For any positive number p and any algebraic expression X: a) The solutions of $\lvert X\rvert < p$ are those numbers that satisfy $-p < X < p$. b) The solutions of $\lvert X\rvert > p$ are those numbers that satisfy $X < -p$ or $X > p$. Similar statements hold for inequalities with $\leq$ or $\geq$.	Solve: a) $\lvert 5x+3\rvert < 2$; b) $\lvert x+3\rvert \geq 1$. a) $\lvert 5x+3\rvert < 2$ $-2 < 5x+3 < 2$ $-5 < 5x < -1$ $-1 < x < -\frac{1}{5}$ The solution set is $\left\{x \middle\vert -1 < x < -\frac{1}{5}\right\}$, or $\left(-1, -\frac{1}{5}\right)$. b) $x+3 \leq -1 \ \textit{or}\ x+3 \geq 1$ $x \leq -4 \ \textit{or}\ x \geq -2$ The solution set is $\{x \vert x \leq -4 \ \textit{or}\ x \geq -2\}$, or $(-\infty, -4] \cup [-2, \infty)$.	22. Solve: $\lvert 3x+2\rvert > 8$. A. $(-\infty, -2) \cup (2, \infty)$ B. $\left(-\infty, -\frac{10}{3}\right) \cup \left(-\frac{10}{3}, \infty\right)$ C. $\left(-\infty, -\frac{10}{3}\right) \cup (2, \infty)$ D. $\left(-\frac{10}{3}, 2\right)$